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ABSTRACT

The books in this series provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. They aim to guide the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The main objective of the activities in this book is for the student to be able to build any design of the pattern. The activities allow students to use pattern blocks to build and analyze the given design, then determine the relationship between the number of the design and the number of blocks necessary to build the design. Exploration and discovery of individual solution strategies are encouraged. The book is laid out in an easy-to-use format with 50 pattern pages (divided into five sections), an appendix (including teaching guide and a glossary), and an answer key. Within the five sections included, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. (JRH)

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Building Algebraic Thinking with Progressive Patterns

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PATTERN BLOCKS

Building Algebraic Thinking with Progressive Patterns

Bob Willcutt

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ABOUT THE PROGRAM

Goals

The books *Building Algebraic Thinking with Progressive Patterns* (*CUBES*, *RODS*, and *PATTERN BLOCKS*) provide an essential ingredient in the transition from concrete mathematical experience to the symbolic reasoning used in algebra. The program guides the student in building and extending manipulative patterns, collecting numerical data, and analyzing the results. The activities engage creativity and provide positive early experiences with a variable, helping students make a smooth transition into the study of algebra. *PATTERN BLOCKS* focuses on the use of pattern blocks (varied shapes); *RODS*, on Cuisenaire-type rods; and *CUBES*, on cubic blocks. The books are self-contained and do not need to be used in any special order.

Building Algebraic Thinking is a discovery-based approach, using a questioning strategy to provide problem solving experiences as recommended by the National Council of Teachers of Mathematics (N.C.T.M.). By following the given strategy and building patterns with manipulative materials, students develop the skills needed to

- substitute for a variable in an equation
- write generalized equations using a variable
- find creative solutions based on geometric reasoning
- combine like variables and numbers
- factor
- reverse process

Throughout the book, exploration and discovery of individual solution strategies are encouraged. Ultimate success is attained when students can create and solve designs of their own.

Objective/Methods

On each activity page, the objective is for the student to be able to build any design of the pattern. To do this, the student uses pattern blocks to build and analyze the given designs and then determines the relationship between the number of the design and the number of blocks necessary to build the design. As the pattern progresses from the first design (design number 1) to the 4th and 5th designs, students will notice that the number of blocks required to build each successive design increases in a predictable way. This results in a "generalization," a general equation, using the variable n, that applies to all designs of the pattern. The design number is substituted for n to get the number of blocks needed to build the design. For example, on page 1, the student finds that multiplying the design number by two gives the number of blocks needed to build that design of the pattern (design 3 requires $3 \times 2 = 6$ blocks). Substituting n for the design number, the student can give a generalization of $n \times 2 =$ number of blocks needed. To find the number of blocks needed for building design 12, substitute 12 for n to get $12 \times 2 =$ 24 blocks.

The manipulative activities program can be used with individuals, small groups, and whole classes. Questions on the activity, or pattern, pages are lettered sequentially, but the suggested questions do not have to be answered in the given order. There is no "right" way for students to come to the correct generalization, and they should be encouraged to find their own methods.

Based on the columns on the Number of Blocks page, students may recognize a pat-

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tern of regularly increasing numbers. If the numbers always grow by 2, the student puts that number before n to get $2n$ for the total number of blocks. This "constant difference" method is a systematic and efficient tool for solving even the most challenging generalizations.

On the other hand, "visual" learners may best see the generalization by relating the physical, or spatial, arrangements of blocks to the total number of blocks. If there are 2 blocks in each of 5 rows for design 5, there are $2n$ blocks in all. If the generalization is not readily seen in the given arrangement, students may want to rearrange blocks to make the generalization more obvious.

Some students find the generalization by the constant difference method and then explain how it is verified by the block arrangement. Enlisting the creative imagination (What does this design remind you of?) can enhance the learning process. Whatever methods are used, it is important that students share ideas and see that there can be a variety of creative ways to find generalizations.

Using the Book

Building Algebraic Thinking is laid out in an easy-to-use format with 50 pattern pages (divided into 5 sections), an appendix (including 4 reproducible Number of Blocks pages, a teaching guide, and a glossary), and an answer key. Each pattern includes a progression (numbered 1 through n) of designs, each consistent with the overall pattern. All pattern problems involve one variable (n) and require one, two, three, four, or even five different types of manipulative objects (shown as shapes representing pattern blocks). The problems become increasingly more complex in each of the book's five sections. Answers are found in the answer key at the end of the book.

Each student (or cooperative group) should have one Number of Blocks page for each pattern page and enough blocks to build as many designs as necessary; the number of designs built may vary with the students. (See the appendix for reproducible Number of Blocks pages.) Following the illustrated pattern as a model, students use the blocks to build the designs; the designs can be used to generate the number data for the Number of Blocks page. For each design built, students must record the numbers of blocks needed. This task reinforces the importance of collecting correct data, as do many science-based activities, because one wrong entry will break the emerging pattern!

The questions on each pattern page help direct students in analyzing relationships between the numbers of the designs and the numbers of blocks to determine and express a generalized equation using the variable n. To foster students' experience with a variable, you might ask, "What would happen if I decided to use n to stand for any number? The n could be 10 or 580 or whatever we want it to be. What will we put in the blocks column if we use n for the design number?" (See chart at right.) The answer will be $2n$, or $n + n$.

Design	
1	2
2	4
3	6
4	8
5	10
⋮	⋮
10	20
⋮	⋮
20	40
⋮	⋮
n	2n

Students apply the generalization to determine the number of blocks required for any design of the pattern. Both forward processing (design 2 takes how many blocks?) and reverse processing (which design will take 20 blocks?) are required. Even if the students do not yet have the tools for formal algebraic operations, by exploring with concrete materials, students form a foundation for understanding later algebraic operations.

Within the five sections of the book, patterns generally progress from the simple arrangements that build confidence to the more complex and creative patterns that inspire students to combine their imagination and their mathematical skills. Some of the final exercises (exploring quadratic solutions) may seem very difficult, but students may use calculators to aid exploration (trial and error is encouraged). Though some students

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will no longer actually need to build the patterns after a few pages, others may continue to need the concrete experience into the final section.

Because *Building Algebraic Thinking* is a discovery-based program, it is recommended that teachers use a flexible approach, gearing the program to students' abilities. For those who feel the need for specific guidance, however, a questioning guide is provided in the appendix.

Specific information about each of the five sections can be found in the chart below.

CONTENTS OF MANIPULATIVE ACTIVITIES PAGES

	Manipulatives	Design Formats	Solution Types	Skills/Features	Suggestions
SECTION 1 (Pages 1–10)	Pattern blocks: 1 specified type of block per page	Simple, regular designs	Linear (simple, then extended, as in 3n – 1)	Basic multiplication Reverse processing Building confidence (mental arithmetic, class discussion, exploration)	• For those who need concrete experiences, actually building the designs is crucial • Encourage exploration of the relationship between physical arrangement and generalization • Let students themselves discover the relationship between constant difference and coefficient of "n" • Extensions: have students (1) create own patterns and (2) graph data on an "xy" graph
SECTION 2 (Pages 11–20)	Pattern blocks: 2 specified types of blocks per page	Simple to creatively complex designs	Linear	(See above) Combining like terms	
SECTION 3 (Pages 21–30)	Pattern blocks: 2 specified types of blocks per page	Simple to creatively complex designs	Linear	Combining like terms Reverse processing (intensifies in Section 4) Constant difference method Seeing a connection between the constant difference and the coefficient Overall increase in difficulty (especially pages 34–40)	• Continue to encourage analysis of physical arrangement • Give the first 3 designs of a creative pattern and have students build next (some may come up with a divergent, though still consistent, design); discuss the question, When is a pattern clearly established? • Be sure students succeed with easier patterns before tackling the more difficult
SECTION 4 (Pages 31–40)	Pattern blocks: 3–4 specified types of blocks per page	Simple to creatively complex designs	Linear		
SECTION 5 (Pages 41–50)	Pattern blocks: 1–3 specified types of blocks per page	Simple to creatively complex designs	Quadratic (very challenging by pages 48–50)	Combining like terms Reverse processing Combining creativity and mathematics skills Many students may not need the building experience; some may need an intermediate step to the abstract As spatial method becomes more difficult, constant difference becomes more dominant Students see that the constant difference divided by 2 equals the coefficient of "n²" in the answer Students see the relationships between the shapes of designs and the number patterns (squared, triangular, rectangular)	• For a good spatial activity similar to those on standardized tests, show the first three designs of a pattern and have students create the next one • Encourage analysis of both physical arrangement and constant difference • Extensions: have students create their own patterns

Manipulatives

It is very important that students see how the manipulative designs generate the associated numerical pattern; therefore, they must actually build the designs before going on to the number work. Not all students will continue the manipulative work throughout the book, but for those who need concrete experiences (and even adults do!), building and extending the designs is crucial and will provide the primary benefit. Students with special needs get a lot of satisfaction from continually extending the patterns. Students can always return to the numerical work at a later date.

Each student should have access to enough blocks to build the designs. Pattern blocks are available from commercial manufacturers, but if necessary, materials close at hand may be substituted: tiles, cubes, cardboard shapes, paper clips, beans, recycled

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materials, and chips are all possiblilities. The main thing is to have enough manipulatives for students to do the actual building.

The designs used in this book are based on pattern blocks which conform to the characteristics listed at right. You may want to make this information available for students' reference. Manufacurers may refer to the shapes as hexagon, square, triangle, trapezoid, parallelogram, and rhombus, respectively. However, the last two shapes are actually both parallelograms, and they are also both rhombuses. For ease of communication, you may want to distinguish the different blocks by color and have the students do the same. (For suggestions on labelling columns on the Number of Blocks pages, see page 54.)

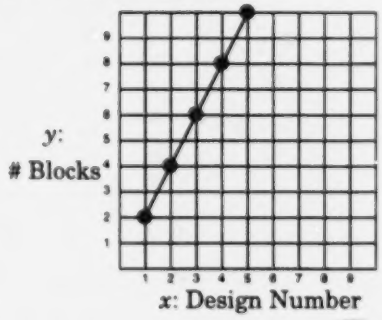
Pattern Block	Color
⬡	Yellow
□	Orange
△	Green
⏢	Red
▱	Blue
◇	Tan

Extension Activities

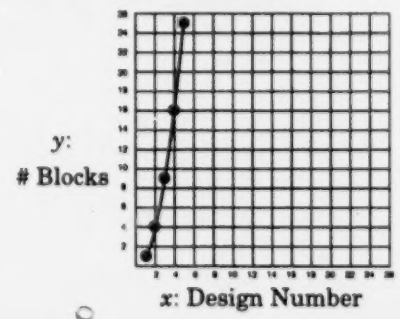
The activities below can provide a valuable extension to the regular manipulative activities.

- Creating patterns: Have students make their own designs adhering to the criteria for a pattern. Remind them that progressive designs must be consistent with the overall pattern; patterns must be developed to the point where, based on the given designs, there is only one subsequent design that can logically follow. Have students share their own patterns with other students or their families and challenge them to solve for the generalization. A copy of the pattern will make an appropriate entry in a student's learning portfolio. Also, each student could contribute one pattern page to a class pattern book.
- Graphing the equations: Give students graph paper and instruct them to label and mark the x-axis with design numbers and the y-axis with the number of blocks used. Have them plot the number of blocks for each of a specified number of designs and join the points. Guide them in recognizing the relationship of the line (straight or curved) to the type of solution required for the generalized equation (linear or quadratic). See graphing examples below.

Linear Function Graph



Quadratic Function Graph



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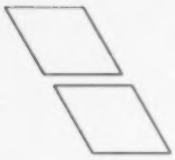
PATTERN 1

1



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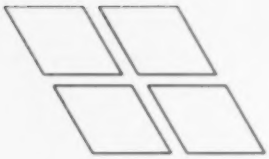

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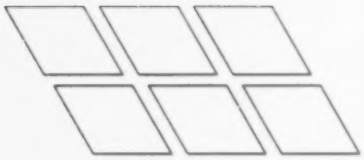

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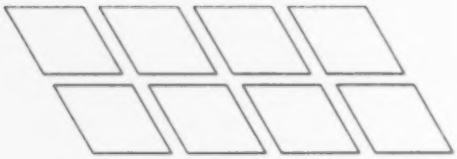

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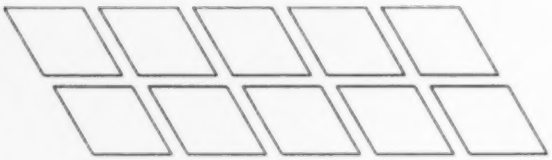

⋮

10

⋮

20

⋮

n

A. How many ◺s will we need for the 18th design? _____

B. After filling in the generalization for design *n* (the last blank) on the Number of Blocks page, look at how the blocks are arranged. Explain one way the generalization is shown in the block arrangements.

_____

_____

_____

C. With 70 ◺s, we can build the _____ design.

D. Will we ever use exactly 83 ◺s to build a design? _____ Why?

_____

_____

_____

E. We will need _____ ◺s to build the 97th design.

F. Which design can we build with 258 ◺s?

_____

G. Examine the numbers you recorded in the ◺ column on the Number of Blocks page. By how much do the numbers grow from one design to the next? _____

How does that growing number relate to the generalization for *n* at the bottom of the column?

_____

_____

_____

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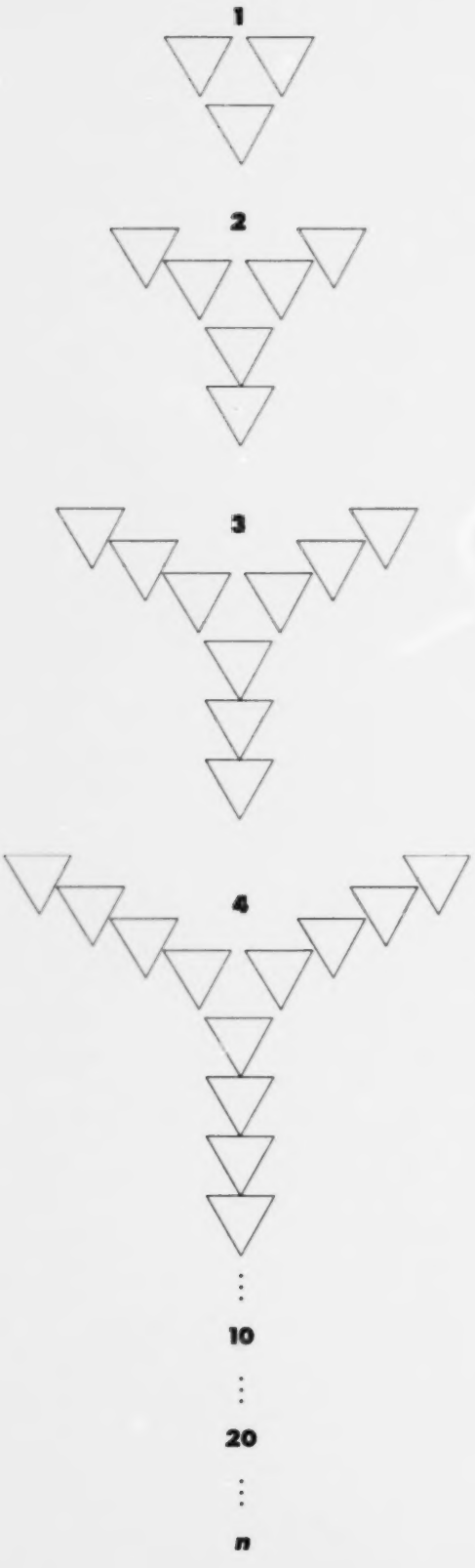
PATTERN 2



A. How many △s will we need for the 24th design? _____

B. Study the block designs; explain one way the arrangements show the generalization that you wrote for design *n* on the Number of Blocks page.

_____

_____

_____

C. With 69 △s, we can build the _____ design.

D. Will we ever use exactly 86 △s to build a design? _____ Why?

_____

_____

_____

E. We will need _____ △s to build the 79th design.

F. Which design can we build with 258 △s? _____

G. Examine your numbers on the Number of Blocks page. By how much do the numbers grow in the △ column? _____

How does that growing number relate to the generalization for design *n* at the bottom of the column?

_____

_____

_____

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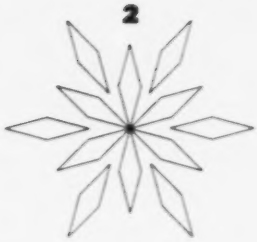
PATTERN 3

1

2



3

4

⋮

10

⋮

20

⋮

n

A. How many ◇s will we need in order to create the 17th design? _____

B. Explain one way you can see the generalization for design *n* by studying the arrangement of blocks.

_____

_____

_____

C. With 132 ◇s, we can build the _____ design.

D. Will we ever use exactly 192 ◇s to build a design? _____ Why?

_____

_____

_____

E. We will need _____ ◇s to build the 53rd design.

F. With 402 ◇s, which design can we build? _____

G. By what amount do the numbers grow in the ◇s column on the Number of Blocks page? _____

How does that growing number, or constant difference, relate to the generalization for *n*?

_____

_____

_____

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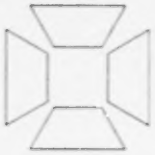
PATTERN 4

1



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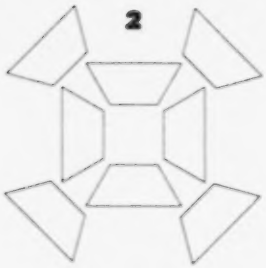

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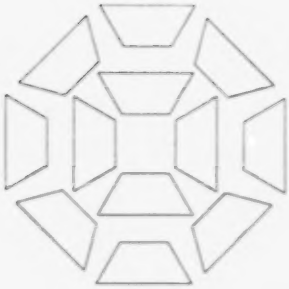

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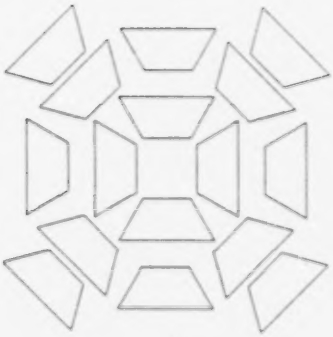

⋮

10

⋮

20

⋮

n

A. How many ⏢s will we need for the 23rd design? _____

B. Explain one way the block arrangements demonstrate the generalization for *n*.

_____

_____

_____

C. With 112 ⏢s, we can build the _____ design.

D. Will we ever use exactly 154 ⏢s to build a design? _____ Why?

_____

_____

E. How many ⏢s will the 74th design require? _____

F. With 356 ⏢s, we can build the _____ design.

G. By how much do the numbers grow in the ⏢ column on the Number of Blocks page? _____

How does that growing number, or constant difference, relate to the generalization for *n*?

_____

_____

_____

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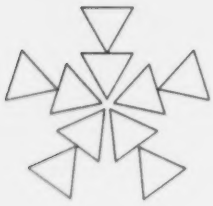
PATTERN 5

1

2



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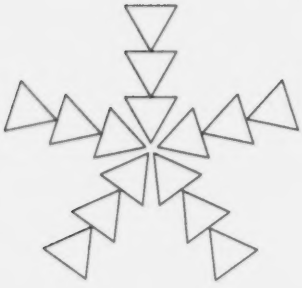

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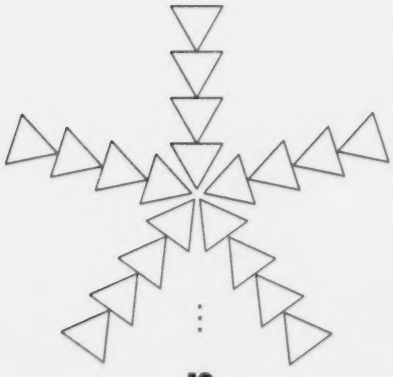

10

20

n

A. For the 27th design, how many △s will we need? _____

B. Study the block designs and explain one way by which the arrangements show the generalization for design n.

_____

_____

_____

C. With 155 △s, we can build the _____ design.

D. Will we ever use exactly 350 △s to build a design? _____ Why?

_____

_____

_____

E. How many △s will be needed to build the 87th design? _____

F. With 480 △s, we can build the _____ design.

G. By how much do the numbers grow in the △ column on the Number of Blocks page?

_____

How does that constant difference relate to the generalization for n?

_____

_____

_____

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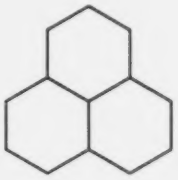
PATTERN 6

1



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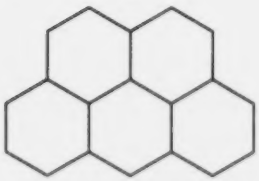

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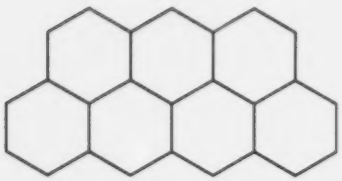

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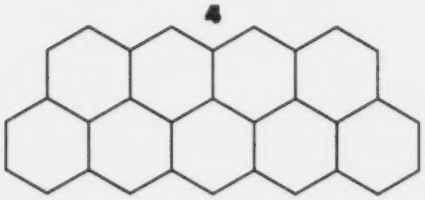

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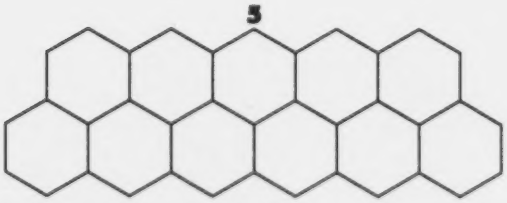

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n

A. For the 25th design, we will need _____ ⬡s.

B. Study the block arrangements and explain one way the designs can show the generalization for the ⬡s.

_____

_____

_____

C. With 139 ⬡s, we can build the _____ design.

D. The generalization for this page ($2n + 1$) is almost the same as the generalization for page 1 ($2n$). Explain how this similarity is shown by the arrangements of blocks.

_____

_____

_____

E. We will need _____ ⬡s to build the 79th design.

F. With 397 ⬡s, which design can we build?

_____

G. By how much do the numbers grow in the ⬡ column on the Number of Blocks page?

_____

How does that constant difference relate to the generalization for n?

_____

_____

_____

14

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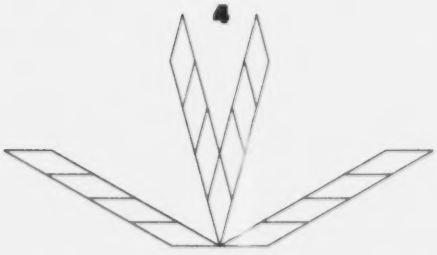
PATTERN 7

1

2

3

4



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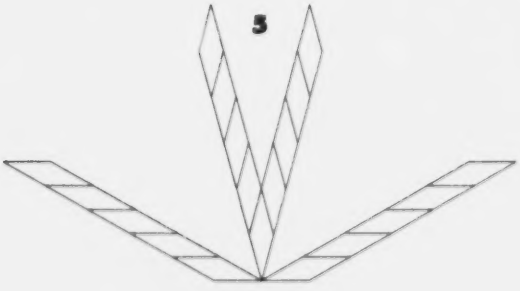

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n

A. We will need ____ ◇s for the 29th design.

B. Study the block designs and explain one way the arrangements give the generalization for the *n*th design.

_____

_____

_____

C. With 139 ◇s, we can build the ____ design.

D. Will we ever use exactly 161 ◇s to build a design? ____ Why?

_____

_____

_____

E. The 4th design uses 7 ◇s for the center of the design. The 10th design will use ____ ◇s for the center of the design.

F. We will need ____ ◇s to build the 88th design.

G. With 371 ◇s, we can build the ____ design.

H. By how much do the numbers grow in the ◇ column on the Number of Blocks page? ____

How does that constant difference relate to the generalization for *n*?

_____

_____

_____

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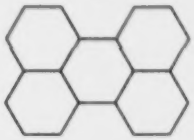
PATTERN 8

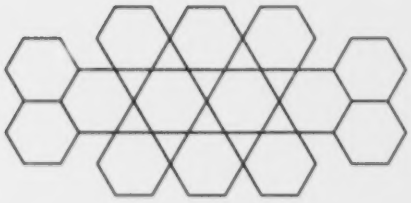
1



2

3

4



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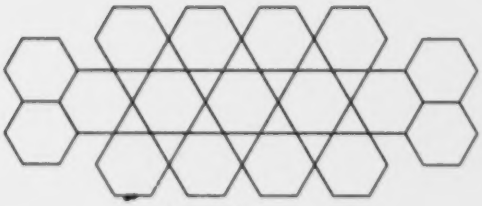

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n

A. How many ⬡s will we need for the 27th design? _____

B. Study the block designs, and explain one way the block arrangements show the generalization for design *n*.

_____

_____

C. With 131 ⬡s, we can build the _____ design.

D. Will we need an even number or an odd number of blocks for the 79th design? _____ Why?

_____

_____

E. Use the pattern in the chart below to answer the following: How many outside blocks will there be for the 28th design? _____ the 77th design? _____

Design	Inside Blocks	Outside Blocks
1	1	4
2	2	6
3	3	8
4	4	10

F. How many ⬡s will we need to build the 86th design? _____

G. With 287 ⬡s, we can build the _____ design.

H. By how much do the numbers grow in the ⬡ column? _____

How does that constant difference relate to the generalization for *n*?

_____

_____

16

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PATTERN 9

1

2

3

4

10

20

n

A. We will need _____ △s for the 29th design.

B. Explain one way the block arrangements show the generalization for the △s for design *n*.

_____

_____

C. With 148 △s, we can build the _____ design.

D. Will we need an even number or an odd number of blocks for the 76th design? _____ Why?

_____

_____

E. The generalization for the △s is almost the same as the generalization found in Pattern 2. How do the design pictures for Patterns 2 and 9 show the close connection between the generalizations?

_____

_____

_____

F. We will need _____ △s to build the 93rd design.

G. With 253 △s, we can build the _____ design.

H. By how much do the numbers grow in the △ column? _____

How does that constant difference relate to the generalization for *n*?

_____

_____

_____

17

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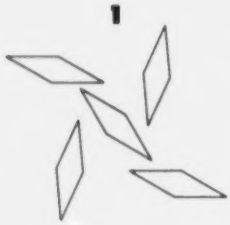
PATTERN 10

1



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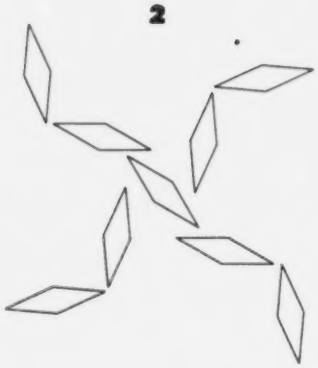

3

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n

A. How many ◇s will we need for the 27th design? ____

B. Study the block arrangements, and explain one way the designs show the generalization for ◇. _____

_____

C. With 149 ◇s, we can build the ____ design.

D. Will we ever use exactly 179 ◇s for a design? ____ Why?

_____

_____

_____

E. By how much do the numbers grow in the ◇ column? ____

How does that constant difference relate to the generalization for n?

_____

_____

_____

F. We will need ____ ◇s to build the 98th design.

G. With 453 ◇s, which design can we build? ____

H. Using the pattern on this page as a model, make a series of designs that show the pattern whose generalization is $5n + 1$.

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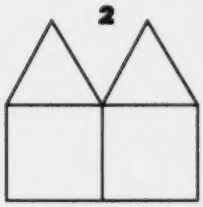
PATTERN 11

1

2



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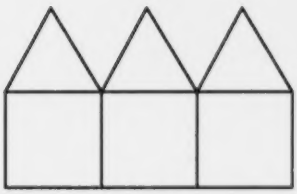

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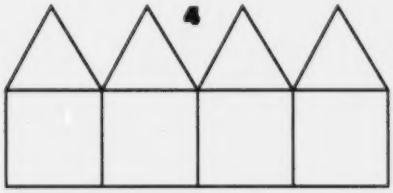

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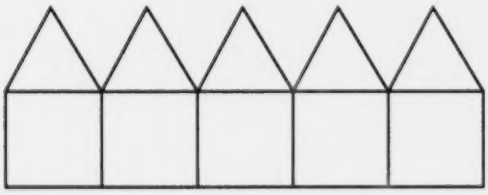

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n

A. We will need _____ □s and _____ △s for the 31st design.

B. Describe the number patterns formed as you fill in the columns on the Number of Blocks page. What kinds of numbers make up the patterns: counting numbers, odd, even, etc.?

□ _____

Total _____

C. When we have a total of 86 blocks, we will be able to build the _____ design.

D. Study the block designs, and explain one way the block arrangements give the generalization for the total number of blocks (the last expression you wrote in the Total column).

_____

_____

_____

E. We will need _____ □s and _____ △s for the 79th design.

F. When we have a total of 292 blocks, which design can we build? _____

G. By how much do the numbers grow in the Total column?

_____

How does that constant difference relate to the generalization for the Total column in design *n*?

_____

_____

_____

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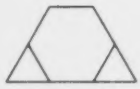
PATTERN 12

1



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n

A. How many ⬡s will we need for the 34th design? _____ How many △s? _____

B. Describe the number patterns formed in the following columns from the Number of Blocks page. What kinds of numbers are these?

⬡ _____

△ _____

Total _____

C. When we have a total of 97 blocks, which design can we build? _____

D. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

_____

E. We will need _____ ⬡s and _____ △s for the 87th design.

F. When we have a total of 303 blocks, we will be able to build the _____ design.

G. By how much do the numbers grow in the Total column? _____

How does that constant difference in the Total column relate to the generalization for design *n* in the Total column?

_____

_____

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20

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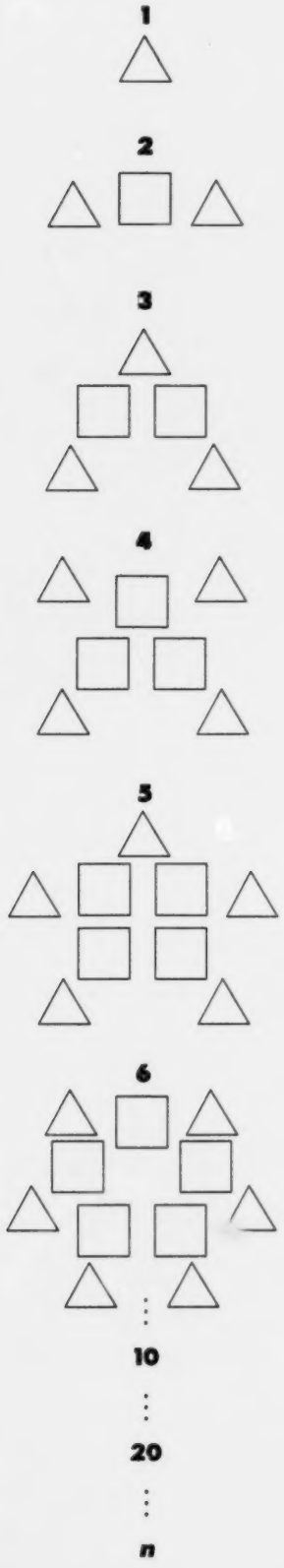
PATTERN 13



A. We will need _____ △s and _____ □s for the 39th design.

B. Which design can we build with a total of 93 blocks? _____

C. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in the Total column (Number of Blocks page)?

_____

How does the constant difference in the Total column relate to the generalization for design *n* in the Total column?

_____

_____

_____

E. When we use 93 □s, how many blocks will we use in all? _____

F. When we use a total of 217 blocks, _____ will be △s and _____ will be □s.

G. Explain how the sum of the generalizations for the △ and □ columns gives the generalization for the Total column.

_____

_____

_____

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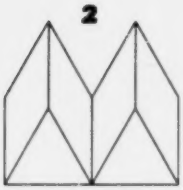
PATTERN 14

1

2



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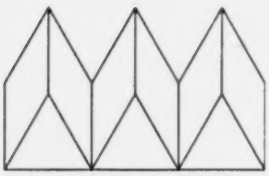

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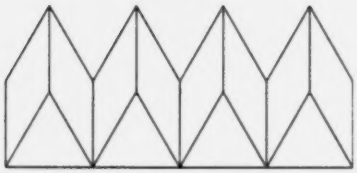

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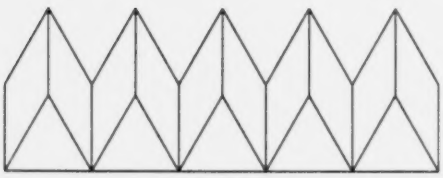

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n

A. For the 37th design, how many △s will we need? _____ How many ◇s? _____

B. Describe the number patterns formed in the following columns from the Number of Blocks page. What do we call the numbers?

△ _____

◇ _____

Total _____

C. When we have a total of 129 blocks, we will be able to build the _____ design.

D. Look at the block designs, and explain one way the block arrangements show the generalization for the total number of blocks.

_____

_____

E. By what amounts do the numbers grow in each of the following columns?

△ _____ ◇ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. Explain how the sum of the generalizations for the △ and ◇ columns gives the generalization for the Total column.

_____

_____

22

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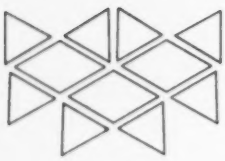
PATTERN 15

1

2



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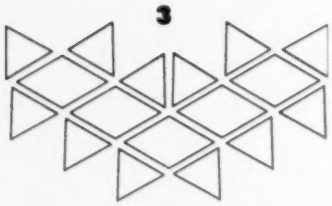

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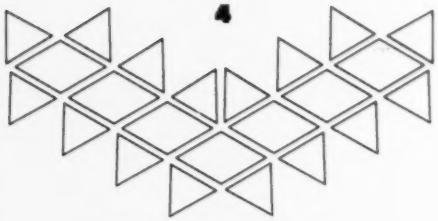

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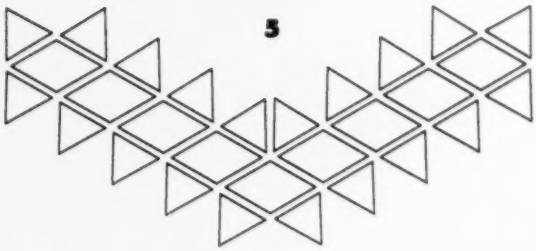

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n

A. We will need ____ ▱s and ____ △s for the 35th design.

B. Give one way the block arrangements show the generalization for the total number of blocks.

_____

_____

C. When we have a total of 281 blocks, we will be able to build the ____ design.

D. Will we ever use exactly 319 ▱s in a design? ____ Why?

_____

_____

Will we use a total of 319 blocks in a design? ____ Why?

_____

_____

E. By how much do the numbers grow in each of the following columns on the Number of Blocks page?

▱ ____ △ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. Explain how the sum of the generalizations for the ▱ and △ columns gives the generalization for the Total column.

_____

_____

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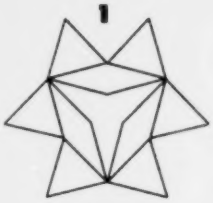
PATTERN 16

1



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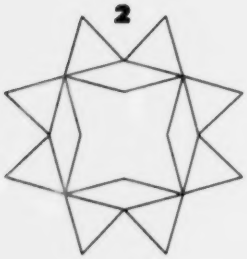
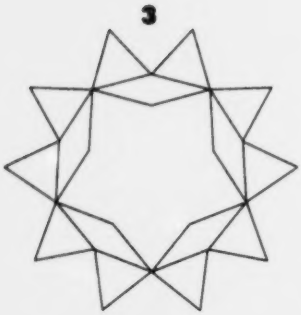


3



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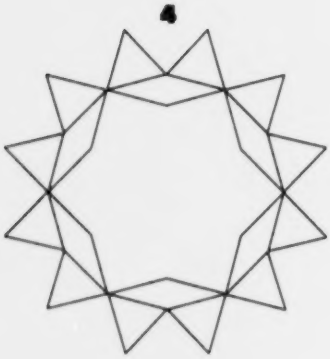

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n

A. We will need ____ ◇s and ____ △s for the 38th design.

B. When we use 44 ◇s, we will need ____ △s.

C. Study the block designs, and explain one way the arrangements demonstrate the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in each column on the Number of Blocks page? ◇ ____ △ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

E. When we use exactly 106 △s, we will use ____ ◇s and a total of ____ blocks.

F. When the total number of blocks is 261, how many will be ◇s? ____

G. Explain how the sum of the generalizations for the ◇ and △ columns gives the generalization for the Total column.

_____

_____

_____

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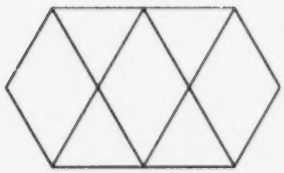
PATTERN 17

1

2

3



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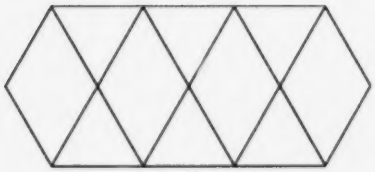

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n

A. We will need ____ ▱s and ____ △s for the 37th design.

B. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

C. Tell whether we will ever use each of the following numbers of blocks and why:

140 ▱s _____

140 △s _____

140 blocks _____

D. By how much do the numbers in each column of the "Number of Blocks" page grow? ▱ ____ △ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

E. When we use exactly 110 △s, we will use ____ ▱s and a total of ____ blocks.

F. How many ▱s will we use when the total number of blocks is 205? ____

G. When the total number of blocks is 280, ____ will be △s.

H. Explain how the sum of the generalizations for the ▱ and △ columns gives the generalization for the Total column.

_____

_____

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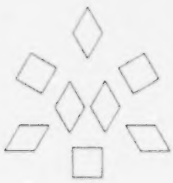
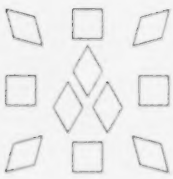
PATTERN 18

1

2



3



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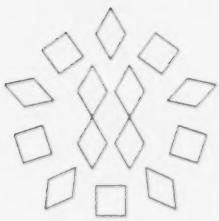

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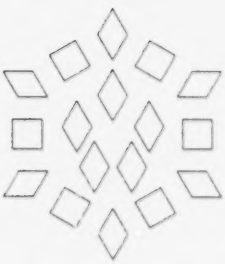

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A. How many □s will we need for the 39th design? ____ How many ◺s? ____

B. How many ◺s will we need when we use 63 □s? ____

C. Study the block designs, and explain one way the block arrangements give the generalization for the total number of blocks.

D. By how much do the numbers grow in each column on the Number of Blocks page? □ ____ ◺ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

E. When we use exactly 139 ◺s, we will use ____ □s and a total of ____ blocks.

F. When the total number of blocks is 245, how many will be ◺? ____

G. Explain how the sum of the generalizations for the □ and ◺ columns gives the generalization for the Total column.

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PATTERN 19

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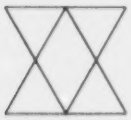

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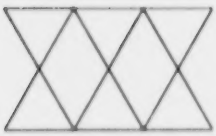

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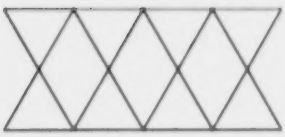

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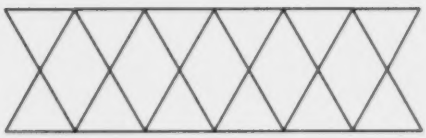

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A. We will need _____ ▱s and _____ △s for the 35th design.

B. How many △s will we need when we use 58 ▱s? _____

C. Look at the block designs, and explain one way they show the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in each column on the Number of Blocks page? ▱ _____ △ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

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E. When we use exactly 126 △s, we will use _____ ▱s and a total of _____ blocks.

F. How many blocks will be ▱s when the total number of blocks is 203? _____

G. When the total number of blocks is 239, _____ will be △s.

H. Explain how the sum of the generalizations for the ▱ and △ columns gives the generalization for the Total column.

_____

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PATTERN 20

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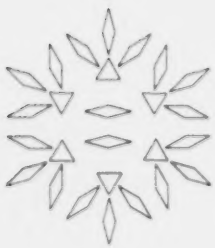

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A. We will need ____ △s and ____ ◇s for the 38th design.

B. When we use 67 △s, we will need ____ ◇s.

C. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in each column on the Number of Blocks page? △ ____ ◇ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

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E. When we use exactly 134 ◇s, we will use ____ △s and a total of ____ blocks.

F. When the total number of blocks is 274, ____ will be △s and ____ will be ◇s.

G. Will the total number of blocks used ever be an odd number? ____ Why?

_____

_____

_____

H. Explain how the sum of the generalizations for the △ and ◇ columns gives the generalization for the Total column.

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PATTERN 21

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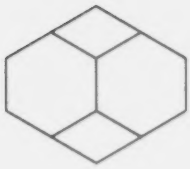

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A. We will need ____ ⬡s and ____ ▱s for the 36th design.

B. How many ▱s will we need when we use 47 ⬡s? ____

C. When we use 80 ▱s, we will need ____ ⬡s.

D. Study the block designs, and explain one way the block arrangements show the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers grow in each column on the Number of Blocks page? ⬡ ____ ▱ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

F. Will the total number of ▱s used ever be an odd number? ____ Why?

_____

_____

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G. When the total number of blocks is 277, ____ will be ⬡s and ____ will be ▱s.

H. Explain how the sum of the generalizations for the ⬡ and ▱ columns gives the generalization for the Total column.

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PATTERN 22

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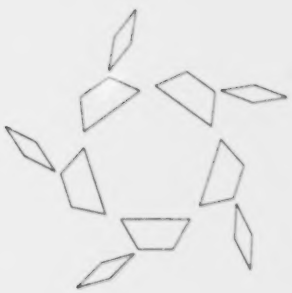

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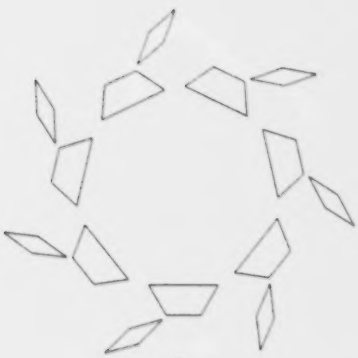

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A. For the 39th design, we will need _____ △s and _____ ◇s.

B. When we use 59 △s, we will need _____ ◇s for a total of _____ blocks.

C. Study the block designs, and explain one way in which the arrangements show the generalization for *n*.

_____

_____

_____

D. By how much do the numbers grow in each column on the Number of Blocks page? △ _____ ◇ _____ Total _____

How does each constant difference relate to the generalization for *n* in its column?

_____

_____

_____

E. Will the total number of blocks used ever be an odd number? _____ Why?

_____

_____

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F. When the total number of blocks is 274, _____ will be △s and _____ will be ◇s.

G. As the design number increases, what shape does the center of our design approach? _____

H. Explain how the sum of the generalizations for the △ and ◇ columns gives the generalization for the Total column.

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PATTERN 23

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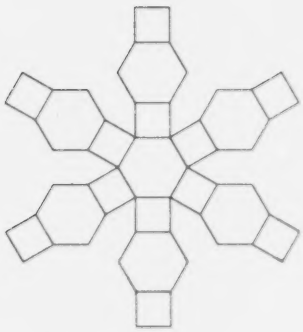

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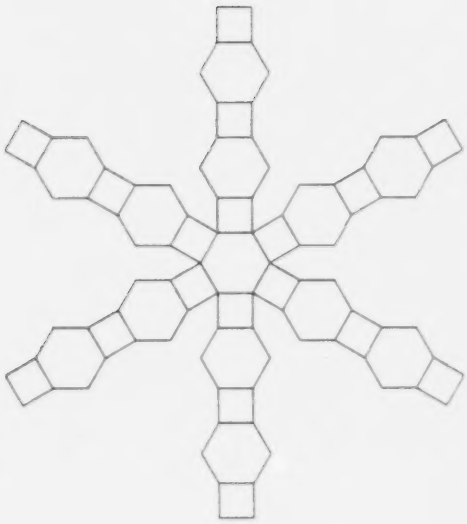

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A. We will need _____ ⬡s and _____ □s for the 35th design.

B. When we use 259 ⬡s, we will need _____ □s for a total of _____ blocks.

C. Describe the number pattern created in the □s column on the Number of Blocks page (what do we call these numbers)?

_____

_____

D. Will we ever use an even number of ⬡s for a design? _____ Why?

_____

_____

E. By how much do the numbers grow in each column on the Number of Blocks page? ⬡ _____ □ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 679, _____ will be ⬡s and _____ will be □s.

G. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

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H. Explain how the sum of the generalizations for the ⬡ and □ columns gives the generalization for the Total column.

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PATTERN 24

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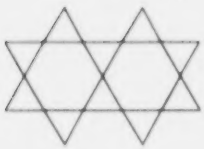

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A. For the 38th design, we will need _____ ⬡s and _____ △s.

B. When we use 182 △s, we will need _____ ⬡s for a total of _____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

272 ⬡s _____

272 △s _____

272 blocks _____

D. Will we ever use an odd number of △s? _____ Why? _____

_____

E. By how much do the numbers grow in each column on the Number of Blocks page? ⬡ _____ △ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. Study the block designs, and explain one way in which the arrangements give the generalization for the total number of blocks.

_____

_____

G. When the total number of blocks is 417, _____ will be ⬡s and _____ will be △s.

H. Explain how the sum of the generalizations for the ⬡ and △ columns gives the generalization for the Total column.

_____

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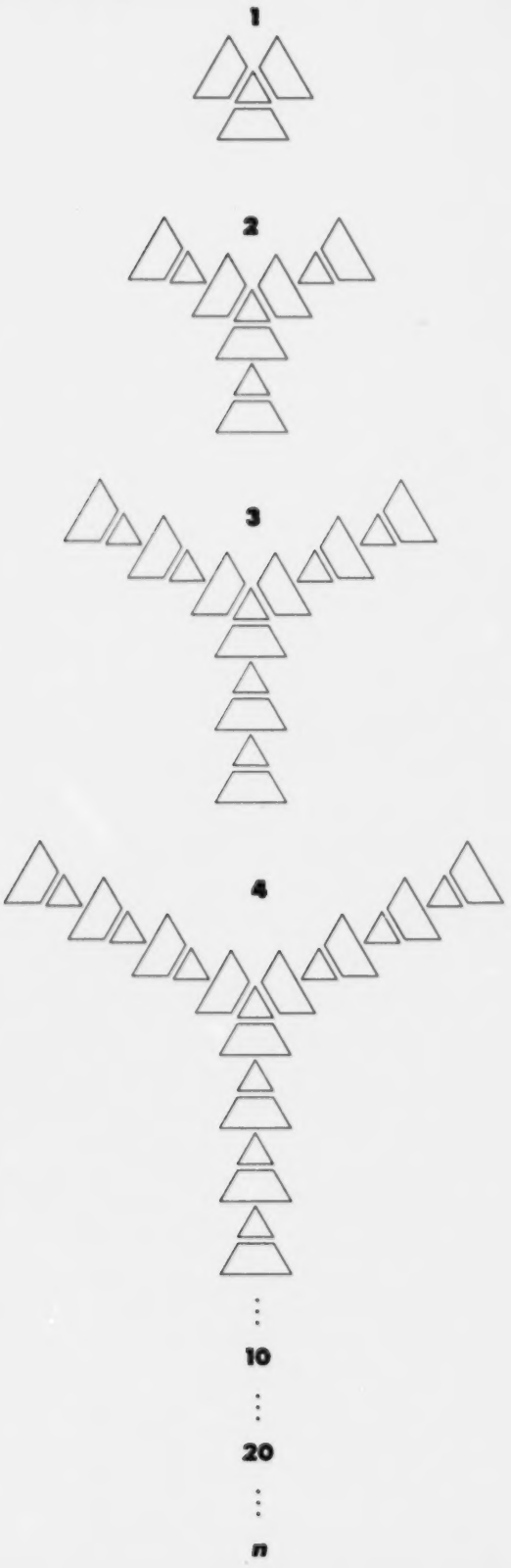
PATTERN 25



A. We will need _____ △s and _____ ⌓s for the 34th design.

B. When we use 142 △s, we will need _____ ⌓s for a total of _____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

316 △s _____

316 ⌓s _____

316 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers grow in each column?

△ _____ ⌓ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

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F. When the total number of blocks is 424, _____ will be △s and _____ will be ⌓s.

G. Explain how the sum of the generalizations for the △ and ⌓ columns gives the generalization for the Total column.

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PATTERN 26

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A. How many △s will we need for the 38th design? ____ How many ◇s? ____

B. When we use 48 △s, we will need ____ ◇s for a total of ____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

243△s _____

243 ◇s _____

243 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers grow in each column?

△ ____ ◇ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 510, ____ will be △s and ____ will be ◇s.

G. Explain how the sum of the generalizations for the △ and ◇ columns gives the generalization for the Total column.

_____

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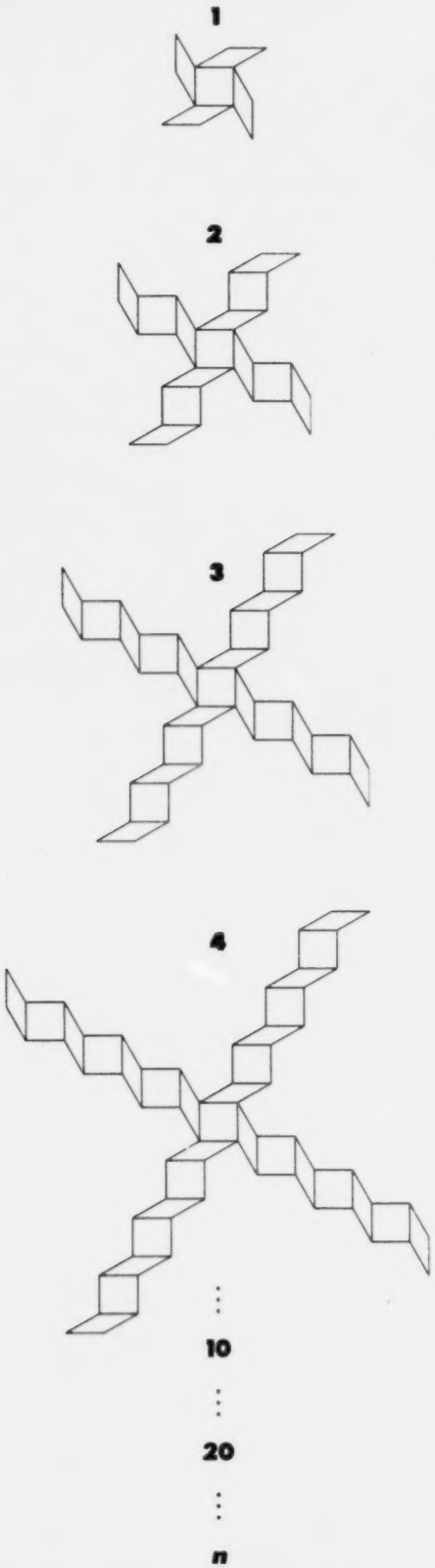
PATTERN 27



A. For the 36th design, we will need ____ □s and ____ ◇s.

B. When we use 201 □s, we will need ____ ◇s for a total of ____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

501 □s _____

501 ◇s _____

501 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers in each column grow?

□ ____ ◇ ____ Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 693, ____ will be □s and ____ will be ◇s.

G. Explain how the sum of the generalizations for the □ and ◇ columns gives the generalization for the Total column.

_____

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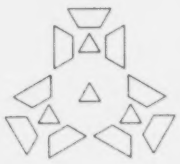
PATTERN 28

1



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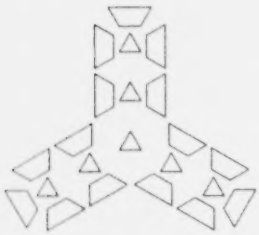

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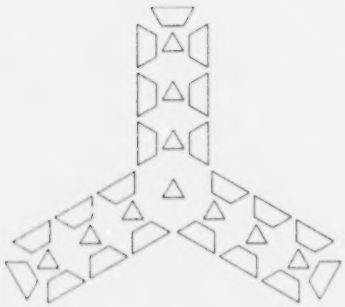

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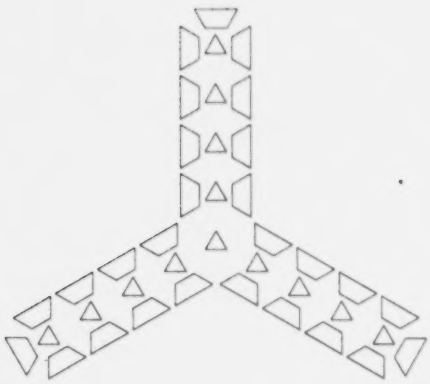

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A. We will need _____ △s and _____ ⌓s for the 38th design.

B. When we use 142 △s, we will need _____ ⌓s for a total of _____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

463△s _____

463 ⌓s _____

463 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers grow in each column?

△ _____ ⌓ _____ Total ___

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 571, _____ will be △s and _____ will be ⌓s.

G. Explain how the sum of the generalizations for the △ and ⌓ columns gives the generalization for the Total column.

_____

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PATTERN 29

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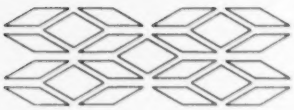
2



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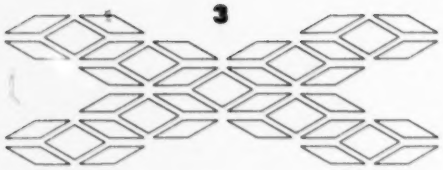

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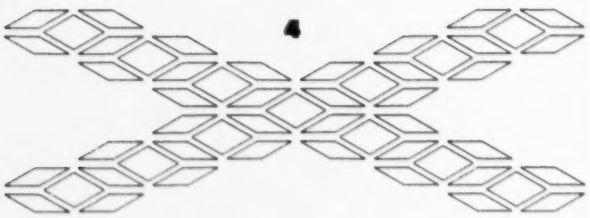

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A. For the 35th design, how many ◇s will we need? _____ How many ◇s? _____

B. When we use 181 ◇s, we will need _____ ◇s for a total of _____ blocks.

C. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in each column?

◇ _____ ◇ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

E. When the total number of blocks is 757, _____ will be ◇s and _____ will be ◇s.

F. Explain how the sum of the generalizations for the ◇ and ◇ columns gives the generalization for the Total column.

_____

_____

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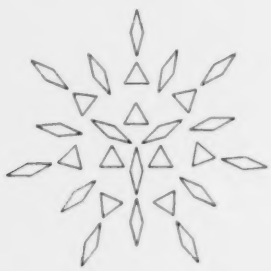
PATTERN 30

1



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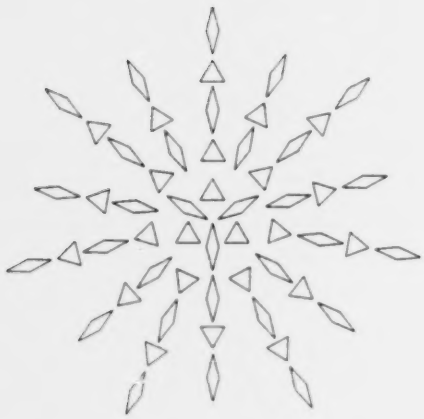

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A. We will need _____ △s and _____ ◇s for the 39th design.

B. When we use 332 ◇s, we will need _____ △s for a total of _____ blocks.

C. Study the block designs, and explain one way the arrangements give the total generalization for *n*.

_____

_____

D. By how much do the numbers grow in each column?

△ _____ ◇ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

_____

E. When we use 374 △s, we will need _____ ◇s for a total of _____ blocks.

F. When the total number of blocks is 951, _____ will be △s and _____ will be ◇s.

G. Explain how the sum of the generalizations for the △ and ◇ columns gives the generalization for the Total column.

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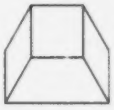
PATTERN 31

1



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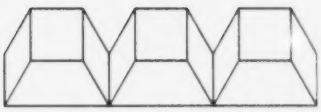

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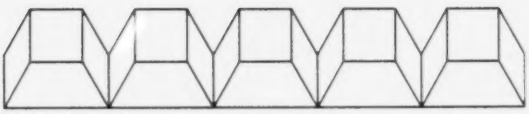

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n

A. We will need _____ ⏢s, _____ ◇s, and _____ □s for the 35th design.

B. When we use 88 ◇s, we will need _____ ⏢s and _____ □s for a total of _____ blocks.

C. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

_____

D. Describe the number patterns formed in the following columns from the Number of Blocks page. What kinds of numbers are these?

◇ _____

_____

Total _____

_____

E. When the total number of blocks is 268, _____ will be ⏢s, _____ will be ◇s, and _____ will be □s.

F. Explain how the sum of the generalizations for the ⏢, ◇, and □ columns gives the generalization for the Total column.

_____

_____

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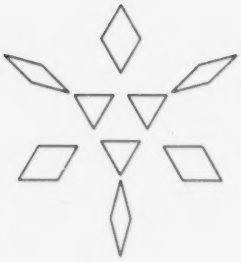
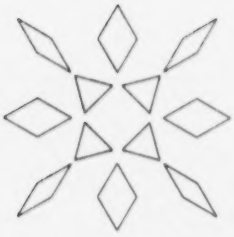
PATTERN 32

1

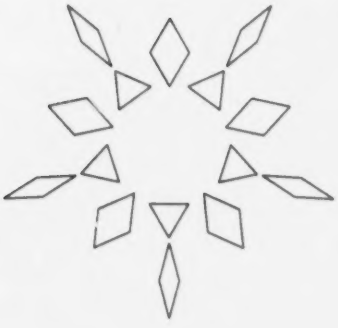
2



3



4



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n

A. We will need ____ △s, ____ ▱s, and ____ ◇s for the 38th design.

B. When we use 46 ▱s, we will use a total of ____ blocks.

C. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

_____

D. When we use 83 ◇s, we will use ____ △s, and the total number of blocks will be ____.

E. Will we ever use a total of 200 blocks? ____ Why?

_____

_____

_____

F. When the total number of blocks is 273, ____ will be △s, ____ will be ▱s, and ____ will be ◇s.

G. Explain how the sum of the generalizations for the △, ▱, and ◇ columns gives the generalization for the Total column.

_____

_____

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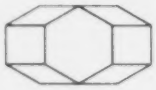
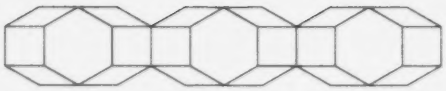
PATTERN 33

1

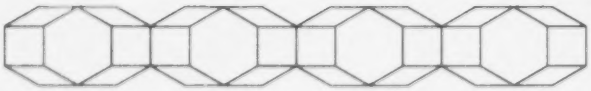


2

3



4



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n

A. For the 37th design, we will need _____ ⬡s, _____ □s, and _____ ◇s.

B. When we use 64 □s, we will use _____ ⬡s and _____ ◇s for a total of _____ blocks.

C. Study the block designs, and explain one way the arrangements show the generalization for the total number of blocks.

_____

_____

_____

D. By how much do the numbers grow in each column?

⬡ _____ □ _____ ◇ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

E. When the total number of blocks is 427, _____ will be ⬡s, _____ will be □s, and _____ will be ◇s.

F. Explain how the sum of the generalizations for the ⬡, □, and ◇ columns gives the generalization for the Total column.

_____

_____

_____

41

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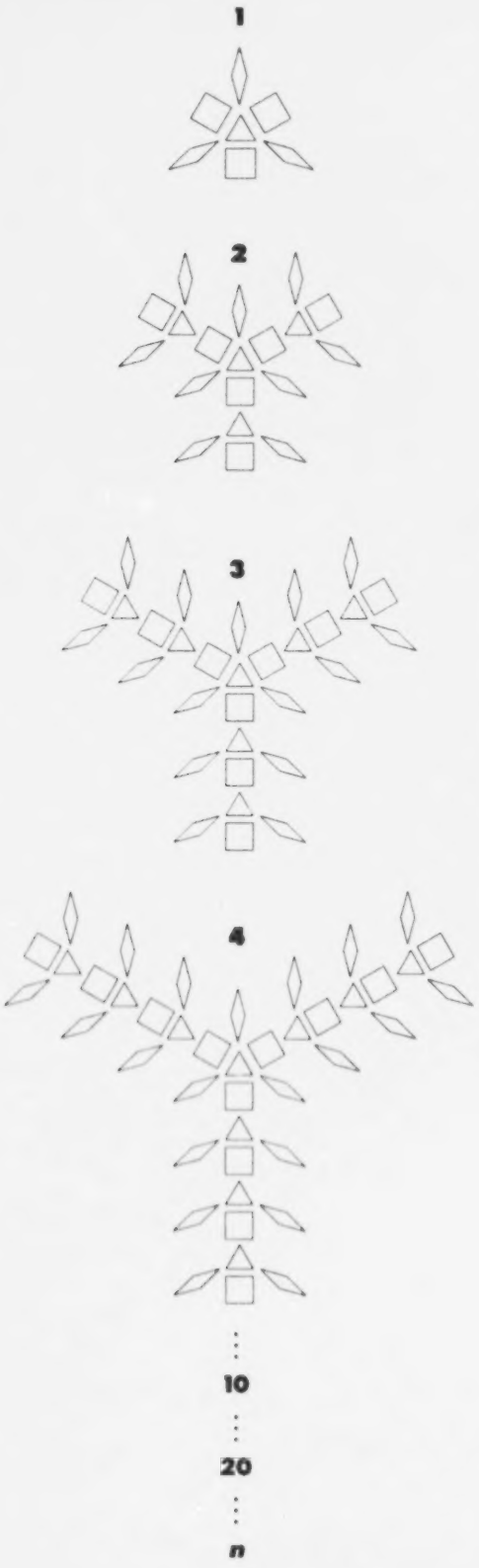
PATTERN 34



A. For design 38, we will need ____ △s, ____ □s, and ____ ◇s.

B. When we use 171 ◇s, we will use ____ △s and ____ □s for a total of ____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

243 △s _____

243 □s _____

243 ◇s _____

243 blocks _____

D. Study the block arrangements and tell one way they show the generalization for the total number of blocks.

_____

_____

E. By how much do the numbers grow in each column?

△ ____ □ ____ ◇ ____ Total ____

How does the constant difference in each column relate to the generalization for n in that column?

_____

_____

F. When the total number of blocks is 559, ____ will be △s, ____ will be □s, and ____ will be ◇s.

G. Explain how the sum of the generalizations for the △, □, and ◇ columns gives the generalization for the Total column.

_____

_____

_____

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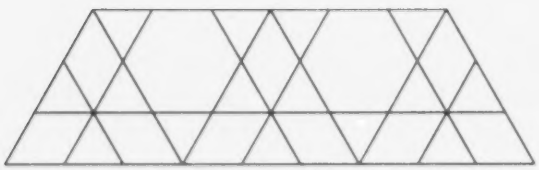
PATTERN 35

1

2

3



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n

A. We will need _____ ▱s, _____ △s, and _____ ⬡s for the 35th design.

B. When we use 249 △s, we will use _____ ▱s and _____ ⬡s for a total of _____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

279 ▱s _____

279 △s _____

279 ⬡s _____

279 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers grow in each column?

▱ _____ △ _____ ⬡ _____ Total _____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 506, _____ will be ▱s, _____ will be △s, and _____ will be ⬡s.

G. Explain how the sum of the generalizations for the ▱, △, and ⬡ columns gives the generalization for the Total column.

_____

_____

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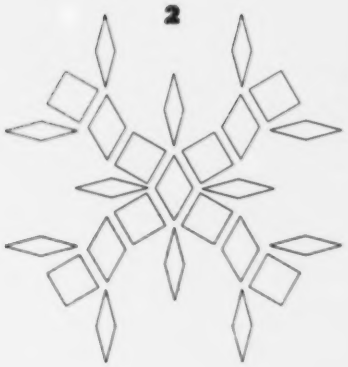
PATTERN 36

1

2



3

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n

A. For the 38th design, we will need ____ ◺s, ____ □s, and ____ ◇s.

B. When we use 180 □s, we will use ____ ◺s and ____ ◇s for a total of ____ blocks.

C. Tell whether we will ever use each of the following numbers of blocks and why:

412 ◺s _____

412 □s _____

412 ◇s _____

412 blocks _____

D. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

E. By how much do the numbers grow in each column?

◺ ____ □ ____ ◇ ____

Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 809, ____ will be ◺s, ____ will be □s, and ____ will be ◇s.

G. Explain how the sum of the generalizations for the ◺, □, and ◇ columns gives the generalization for the Total column.

_____ 44 _____

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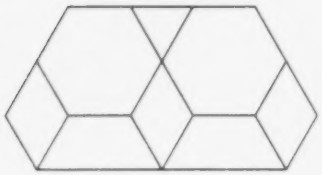
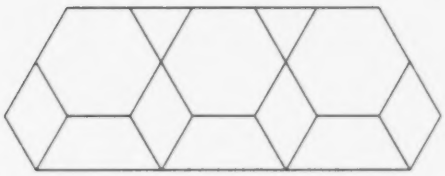
PATTERN 37

1

2



3



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n

A. We will need _____ ⌓s, _____ ▱s, _____ ⬡s, and _____ △s for the 39th design.

B. How many ▱s will we need when we use 76 △s? _____

C. Study the block designs, and explain one way the arrangements give the generalization for the total number of blocks.

_____

_____

_____

D. What will be the total number of blocks when we use 93 ▱s? _____

E. Tell whether we will ever use each of the following numbers of blocks and why:

382 ▱s _____

382 ⬡s _____

382 △s _____

382 blocks _____

F. When the total number of blocks is 852, _____ will be ⌓s, _____ will be ▱s, _____ will be ⬡s, and _____ will be △s.

G. Explain how the sum of the generalizations for the ⌓, ▱, ⬡, and △ columns gives the generalization for the Total column.

_____

_____

_____

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PATTERN 38

1

2

3

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⋮

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n

A. We will need ____ △s, ____ ▱s, ____ ▭s, ____ ◇s, and ____ □s for the 36th design.

B. With 261 ▭s, we will use ____ △s.

C. Explain one way the designs show the generalization for the total number of blocks.

_____

_____

D. Tell whether we will ever use each of the following numbers of blocks and why:

327 △s _____

327 ▱s _____

327 ▭s _____

327 ◇s _____

327 blocks _____

E. By how much do the numbers grow in each column?

△ ____ ▱ ____ ▭ ____ ◇ ____

□ ____ Total ____

How does each constant difference relate to that column's generalization for *n*?

_____

F. When the total number of blocks is 1192, ____ will be △s, ____ will be ▱s, ____ will be ▭s, ____ will be ◇s, and ____ will be □s.

G. How does the sum of the generalizations for the △, ▱, ▭, ◇, and □ columns give the generalization for the Total column?

_____

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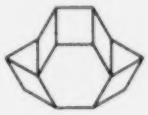
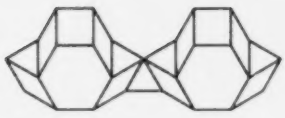
PATTERN 39

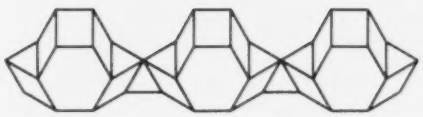
1



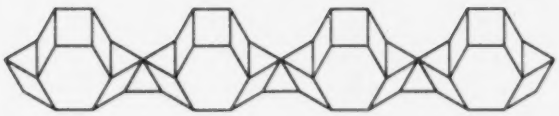
2



3



4



⋮

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n

A. We will need ____ ⬡s, ____ ◇s, ____ □s, and ____ △s for the 38th design.

B. When we use 104 △s, we will use ____ ◇s.

C. Explain one way in which the block arrangements show the generalization for the total number of blocks.

_____

_____

_____

D. Tell whether we will ever use each of the following numbers of blocks and why:

260 ◇s _____

260 △s _____

260 blocks _____

E. By how much do the numbers grow in each column?

⬡ ____ ◇ ____ □ ____ △ ____

Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 512, ____ will be ⬡s, ____ will be ◇s, ____ will be □s, and ____ will be △s.

G. Explain how the sum of the generalizations for the ⬡, ◇, □, and △ columns gives the generalization for the Total column.

_____

47 _____

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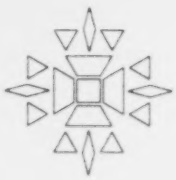
PATTERN 40

1



2

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n

A. We will need ____ □s, ____ ⌓s, ____ △s, and ____ ◇s for the 35th design.

B. When we use 364 ⌓s, how many △s will we need? ____

C. How many □s will we use with 544 ◇s?
____

D. Study the block designs, and explain one way in which the arrangements give the generalization for the total number of blocks.

_____

_____

_____

E. By how much do the numbers in each column grow?

□ ____ ⌓ ____ △ ____ ◇ ____

Total ____

How does the constant difference in each column relate to the generalization for *n* in that column?

_____

_____

F. When the total number of blocks is 1161, ____ will be □s, ____ will be ⌓s, ____ will be △s, and ____ will be ◇s.

G. Explain how the sum of the generalizations for the □, ⌓, △, and ◇ columns gives the generalization for the Total column.

_____

_____

_____

48

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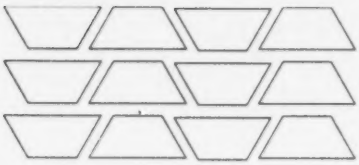
PATTERN 41

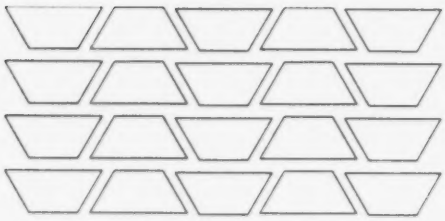
1

2

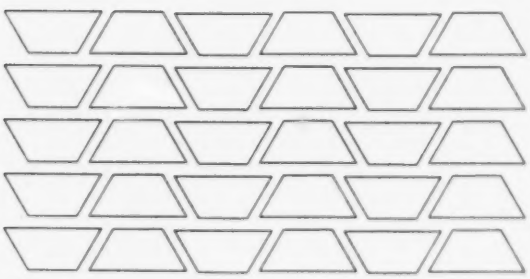
3



4



5



⋮

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n

A. We need _____ ⏢s for the 33rd design.

B. The combined blocks form a basic geometric shape that grows larger with each design. What is the shape called?

_____

C. By what amounts do the numbers grow in the ⏢ column? _____
Is there a constant difference between consecutive numbers? _____ When does a constant difference occur, and what is it?

_____

_____

D. The number of blocks needed for each design is always an even number. Why?

_____

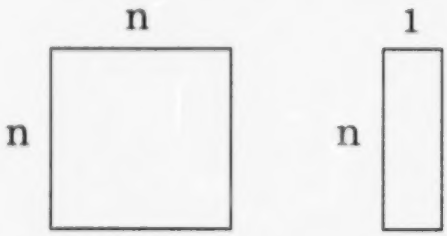
E. When the total number of blocks is 1406, the design number will be _____.

F. How does the generalization for *n* relate to the geometric shape of each design?

_____

_____

G. One student suggested that each design can be rearranged into the shapes of a square and a rectangle, as shown below.



How does this arrangement relate to the generalization for *n*?

_____

_____

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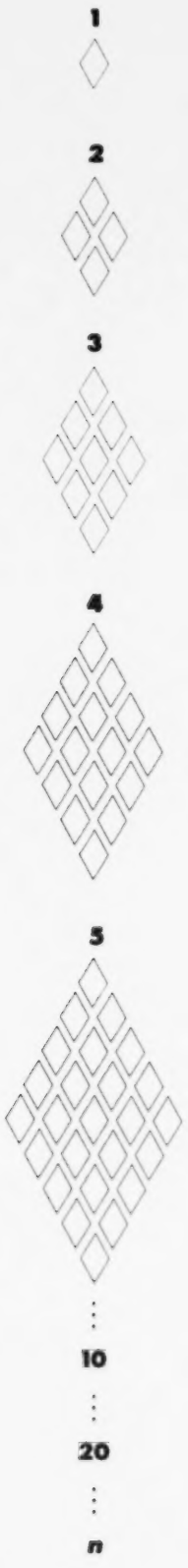
PATTERN 42



A. How many ◺s will we need for the 39th design? _____

B. The numbers created in the ◺ column are a famous sequence of numbers called the square numbers. Can you explain why they have this name?

_____

_____

C. When we use exactly 2209 ◺s, we can create the _____ design.

D. Tell whether we will ever use each of the following numbers of blocks and why:

2601 ◺s _____

3000 ◺s _____

3969 ◺s _____

E. By how much do the numbers grow in the ◺ column? _____

Is there a constant difference between consecutive numbers? _____

When does a constant difference occur, and how much is that difference?

_____

_____

F. One student noted that, for each new design, we add an odd number of blocks (teepee shaped) on the top. The added odd numbers are underlined below.

$\underline{1} = 1$
$1 + \underline{3} = 4$
$1 + 3 + \underline{5} = 9$
$1 + 3 + 5 + \underline{7} = 16$

What will be the sum of the first 48 odd numbers? _____

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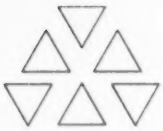
PATTERN 43

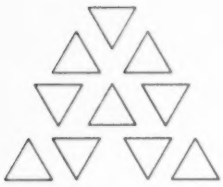
1

2

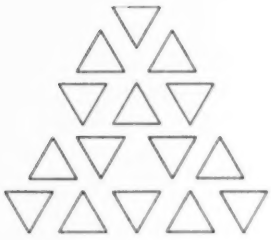
3



4



5



⋮

10

⋮

20

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n

A. We will need _____ △s for design 37.

B. The numbers created in the △ column (Number of Blocks page) are a famous sequence of numbers: the triangle numbers. Why are they called by this name?

_____

C. Triangle numbers can also be represented as below:

Two such similar triangles can be combined to form a rectangle:

How does this arrangement explain the generalization for triangle numbers?

_____

_____

D. By how much do the numbers in the △ column grow? _____

Is there a constant difference between consecutive numbers? _____

When does a constant difference occur, and how much is that difference?

_____

E. Which design will use a total of 1596 blocks? _____

F. One student noted that, to form the series of designs, we add consecutive numbers of blocks, as follows:

$1 = 1$
$1 + 2 = 3$
$1 + 2 + 3 = 6$
$1 + 2 + 3 + 4 = 10$
$1 + 2 + 3 + 4 + 5 = 15$

What will be the sum of the first 50 numbers? _____ 200 numbers? _____ 8000 numbers? _____

51

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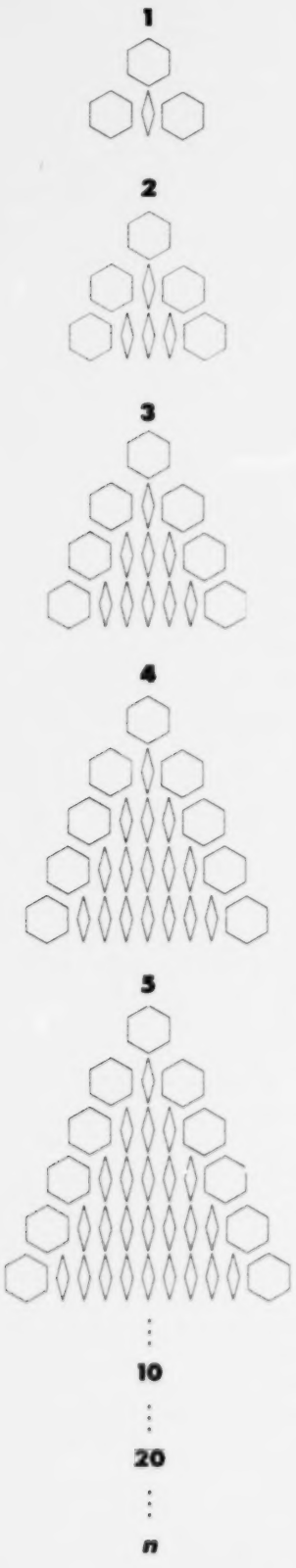
PATTERN 44



A. We will need _____ ◇s and _____ ⬡s for the 35th design.

B. The numbers created in the ◇ column and in the Total column (Number of Blocks page) are a well-known sequence of numbers. What are they called?

_____

Why are they called by this name?

_____

C. By how much do the numbers grow in the ⬡ column? _____

What does that constant difference tell us (where else does it appear)?

_____

D. By what amounts do the numbers grow in the ◇ column? _____

When does a constant difference occur?

_____

What significance does the constant difference have? _____

E. When the total number of blocks is 2209, _____ will be ◇s and _____ will be ⬡s.

F. Explain how the sum of the generalizations for the ◇ and ⬡ columns gives the generalization for the Total column.

_____

_____

G. Some students gave a generalization of $(n^2 + 2n + 1)$ and others gave $(n + 1)^2$. Are these answers the same? _____ Why?

_____

_____

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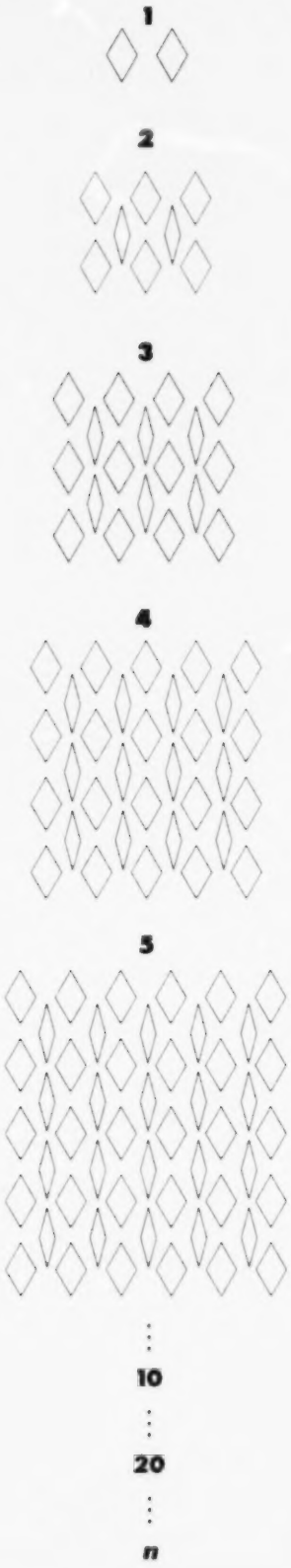
PATTERN 45



A. We will need ____ ◺s and ____ ◇s for the 39th design.

B. One student said that the ◺s by themselves always form a square plus one column of a second square. How does this observation relate to the generalization for the number of ◺s needed?

_____

_____

C. The same student said that the ◇s are always one row short of forming a square. How does this statement relate to the generalization for the number of ◇s?

_____

_____

D. By how much do the numbers increase in each column (Number of Blocks page)?

◺ _____

◇ _____

Total _____

When does a constant difference occur?

_____

What are the constant differences?

_____

What is the significance of the constant difference? _____

_____

E. When the total number of blocks is 6498, ____ will be ◺s and ____ will be ◇s.

F. Explain how the sum of the generalizations for the ◺ and ◇ columns gives the generalization for the Total column.

_____

_____

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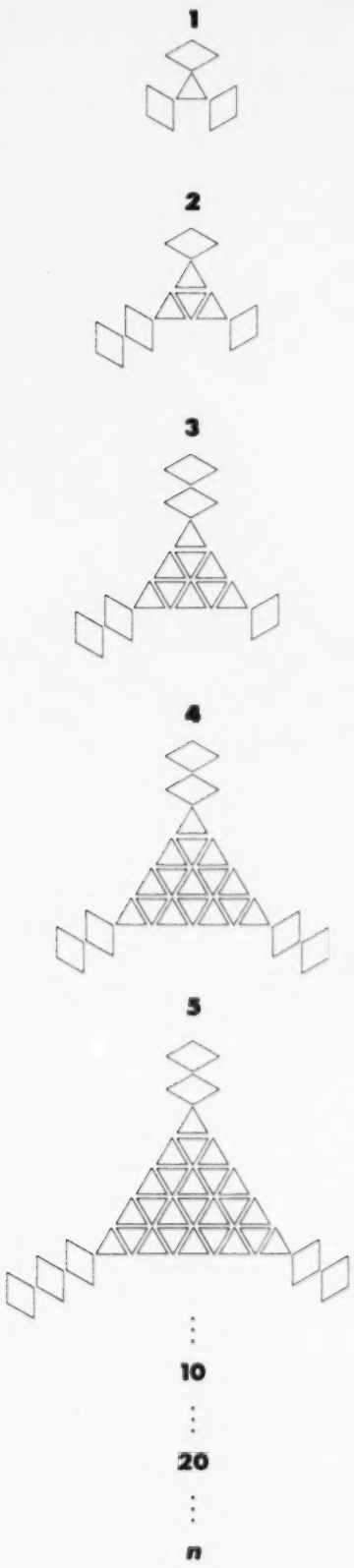
PATTERN 46



A. How many △s will we need for the 38th design? _____ How many ▱s? _____

B. The numbers in the △ column are the famous square numbers. The arrangements of the △s include rows of odd numbers of blocks.

$1 = 1$
$1 + 3 = 4$
$1 + 3 + 5 = 9$

The sum of the first four odd numbers will be 16. What will be the sum of the first 25 odd numbers? _____ 67 odd numbers? _____ 291 odd numbers? _____

C. By how much do the numbers increase in each column?

△ _____

▱ _____

Total _____

When does a constant difference occur, and how much is that constant difference for each column? _____

_____

For the generalization, what is the significance of the constant difference?

_____

_____

D. When we use 67 ▱s, we will need _____ △s.

E. When the total number of blocks is 6008, _____ will be △s and _____ will be ▱s.

F. Explain how the sum of the generalizations for the △ and ▱ columns gives the generalization for the Total column.

_____

_____

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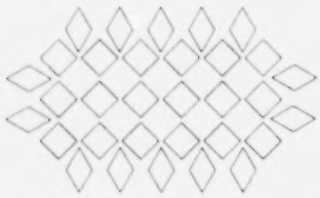
PATTERN 47

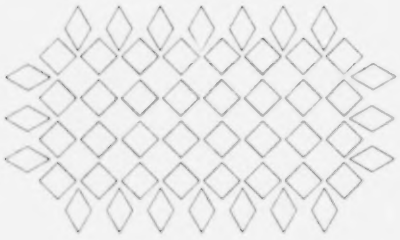
1

2

3



4



5

10

20

n

A. We will need ____ □s and ____ ◺s for the 35th design.

B. A student observed that the □s form the shape of two squares growing side-by-side. How does this observation relate to the generalization in the □s column?

_____

_____

C. By how much do the numbers increase in each column?

□ _____

◺ _____

Total _____

When does a constant difference occur, and how much is that constant difference for each column? _____

_____

For the generalization, what is the significance of the constant difference?

_____

_____

D. How many ◺s will we use with 1682 □s? ____

E. When we have 242 ◺s, we will use ____ □s.

F. When the total number of blocks is 4316, ____ will be □s and ____ will be ◺s.

G. Explain how the sum of the generalizations for the □ and ◺ columns gives the generalization for the Total column.

_____

_____

55

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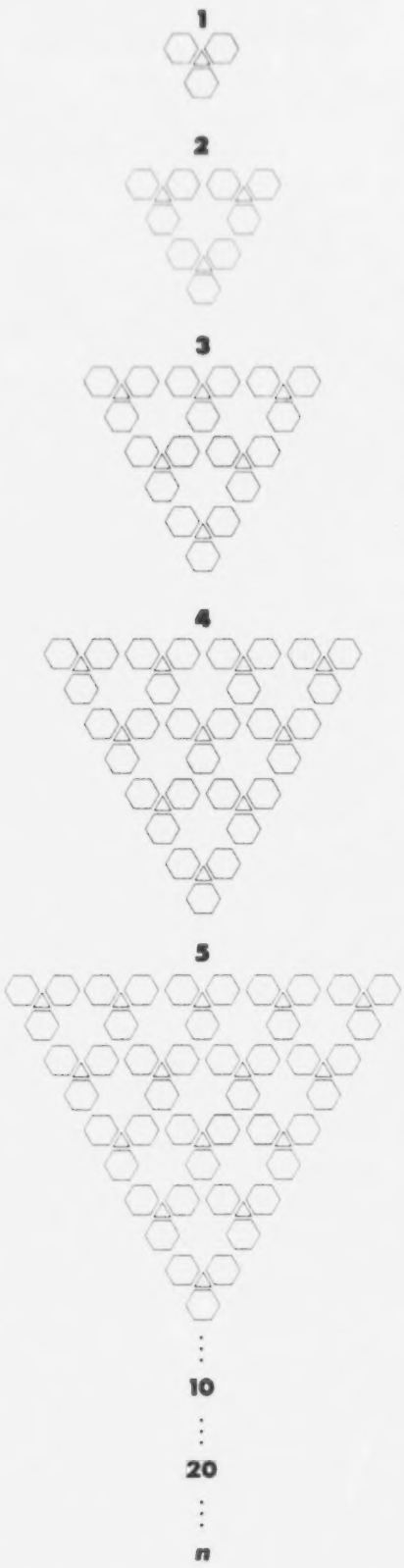
PATTERN 48



A. We will need _____ △s and _____ ⬡s for the 39th design.

B. The △s and the ⬡s each form the shapes of triangles. The numbers represented by the △s are called the triangle numbers. What is the relationship between the numbers of △s and the numbers of ⬡s?

_____

_____

How does this relationship appear in the generalizations?

_____

_____

C. By how much do the numbers increase in each column?

△ _____

⬡ _____

Total _____

When does a constant difference occur, and how much is that constant difference for each column?

_____

Of what significance is the constant difference to the generalization?

_____

_____

D. How many ⬡s will we need when we use 1128 △s? _____

E. When the total number is 6844, _____ will be △s and _____ will be ⬡s.

F. Show how the sum of the generalizations for the △ and ⬡ columns gives the generalization for the Total column.

_____

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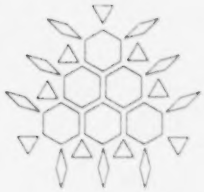
PATTERN 49

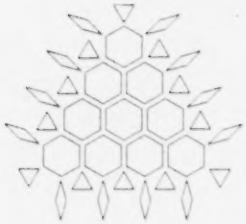
1

2

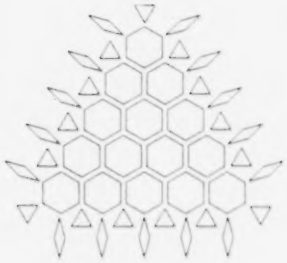
3



4



5



⋮

10

⋮

20

⋮

n

A. We will need _____ ⬡s, _____ △s, and _____ ◇s for the 38th design.

B. Describe the number pattern created in the ◇ column (what do we call these numbers)?

_____

C. In each design, the ⬡s form a basic geometric shape. What is that shape?

_____

What do we call the numbers of blocks forming that shape? _____

D. By how much do the numbers increase in each column?

⬡ _____

△ _____

Total _____

When does a constant difference occur, and how much is that constant difference for each column? _____

_____

For the generalization, what is the significance of the constant difference?

_____

E. When we use 129 ◇s, we will use _____ ⬡s and _____ △s.

F. When we use 1176 △s, we will use _____ ⬡s and _____ ◇s.

G. When the total is 1690, _____ will be ⬡s.

H. Explain how the sum of the generalizations for the ⬡, △, and ◇ columns gives the generalization for the Total column.

_____

_____

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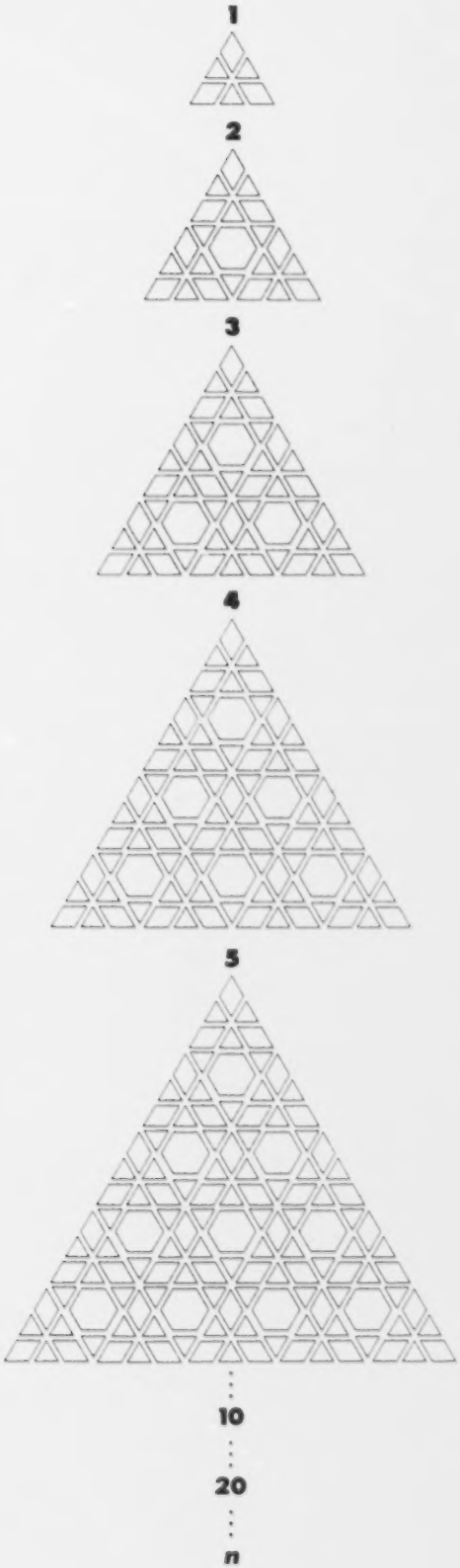
PATTERN 50



A. We will need _____ ▱s, _____ △s, and _____ ⬡s for the 37th design.

B. By themselves, the ⬡s form what basic shape? _____

What do we call the numbers of blocks forming that shape? _____

What is the generalization for the number of ⬡s needed? _____

C. By how much do the numbers grow in each column?

▱ _____

△ _____

Total _____

When does a constant difference occur?

_____

_____

How much are the constant differences for each column, and how can you use them?

_____

_____

_____

D. When we use 588 △s, we will need _____ ▱s and _____ ⬡s.

E. When the total is 2226 blocks, _____ will be ▱s, _____ will be △s, and _____ will be ⬡s.

F. Show how the sum of the generalizations for each type of block equals the generalization for the total number of blocks.

_____

_____

_____

55

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APPENDIX

Glossary of Terms

Building Algebraic Thinking uses a specific vocabulary in providing valuable pre-algebra experiences to the student; it is important that the teacher become familiar with the terms used in the book.

coefficient of *n*: number by which the variable *n* is multiplied

constant difference (growing number): the difference in numbers of blocks used between any two consecutive designs of a pattern

generalization: general equation describing the relationship between the ordinal number of a design (*n*) and the number of blocks used for that design; the "rule" or formula for a pattern

linear function: generalization whose solutions for individual designs increase by a consistent number and whose equation can be plotted as a straight line

pattern: a series of an infinite number of consistent designs in which there is one possible arrangement and number of blocks for each design; the pattern has a "rule" or generalized equation that can be used to determine the numbers of blocks needed in any design

design: arrangement of blocks that represents a stage or progression within a pattern (each activity page shows several designs of one overall pattern); each design requires a specific number and arrangement of blocks consistent with the pattern

quadratic function: generalization involving the square of the variable; it will take on the general form of $ax^2 + bx + c$; graph of this function is a curved line (parabola)

variable: a symbol used as a place holder for a number that can vary in value

reverse processing: solving an equation in which the unknown (variable) is on the left side: instead of finding the number of blocks used in a given pattern design ($2 \times 5 = ?$), the student is given the number of blocks required and must determine the associated design ($2 \times ? = 10$) or the associated number of another kind of block in the design

Teaching Guide

Building Algebraic Thinking is intended to be an exploratory/discovery based program; a formalistic teaching approach is not recommended. However, for those who will gain confidence with some initial structure, the guidelines below provide a questioning strategy that can be used as a model to get you started. Review them before teaching, and as you become familiar with the concepts, rely on your own judgment of what your students need moment-by-moment.

The order of activities for any pattern can be altered as needed: the student may copy and extend the pattern for several designs, fill in the associated numbers of blocks, and answer one or more questions before completing every step in the questioning strategy that follows. As you proceed, keep in mind the following points:

- The steps will familiarize you with the general concepts for the activity pages. In providing problems to the students, you may vary the types and extent of activities on each page; the important thing is that students progress, when they are ready, from the simple concrete experiences to the abstract concepts that naturally evolve.

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- Students may need to just experiment and build many patterns before they are ready to formalize the concepts elicited through any questioning strategy. Be careful not to push them into a stage for which they are not yet ready. It is important for them to get comfortable with each new realization; in the process, don't worry if they seem to "regress" into a previous stage of simple block building.

1. Building the designs

Have the students use their blocks to copy the exact arrangement of objects for the designs shown on the activity page. As they proceed, ask questions to stimulate their analysis of the pattern:

What does this design resemble? How is it growing? What will happen as it grows?

2. Determining the numbers of blocks

Ask students to notice the numbers of blocks used for each pattern design and fill in the blank(s) of the second column on the Number of Blocks page (see p. 54).

How many blocks will we need for the first design? the second? the fifth?

3. Discovering the pattern

Allow the students to discover any number pattern occurring on the Number of Blocks page they have filled out (including odd or even numbers) and the relationship between the design number and the number of blocks used.

How are the numbers of blocks used different from one design to the next? Do you notice any kind of pattern developing in the number of blocks column? How many blocks do we add to each design? How do these "growing" numbers change? Are the differences always the same? Are they odd or even?

When the student can easily see how the numbers change regularly, you may want to use the term **constant difference**.

As the numbers grow, what is the difference occurring between designs? Is there a constant difference—is it the same difference from one design to the next throughout the pattern?

4. Extending the pattern

Encourage the students to "work forward" by having them predict the number of blocks needed for subsequent designs, and then encourage reverse processing by having students predict what design number will require a given number of blocks.

Can you predict how many blocks will be needed in order to build the sixth and seventh designs?

How did you figure out the solution? How is the number of blocks used related to the number of the design?

Which design will require a total of 86 blocks? With 200 blocks, which design could you build?

When the students are able to predict the answers for specific designs, encourage them to explain how they got their answers.

How did you know how many blocks to use for design number 7? What was your formula? (e.g., 2 times the design number or 2×7)

Encourage the students to use mental arithmetic as they determine the rule for generalizing; let different students give the design number and the rest of the class give the end result.

Are there any numbers that will stump us? Can we find the answer for any whole number?

When students can easily give the number of blocks, use n as a variable to represent any design number.

5. Generalizing with a variable

Encourage students to use a variable to create a generalization that can be used to determine the number of blocks for any design in the pattern.

Is there an easy way to find the answer for any design of this particular pattern?

You may want to start using the term *generalize*.

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You've already come up with some specific formulas for the different designs. How about a general formula that applies to the whole pattern? What if we let n *stand for any number? What do we do to* n? (The same as we did to the design number: $2 \times n$ = number of blocks.)

Have students substitute any number for n.

Now, how can we find the number of blocks we need to build the 20th design? What do we do with the number 20? (Substitute for n: $2 \times 20 = 40$ blocks.)

When the generalization is found, it should be checked. Have students go back to the earlier, simpler designs and substitute design numbers for the variable to see that the generalization does indeed work.

6. Analyzing the physical arrangement

It is hoped that students will see that there are many different ways to solve problems and that mathematics is not a one-right-answer discipline. Students may come up with some very creative strategies, and these should be shared.

We have seen that constant difference may help students generalize. Another strategy they may find helpful is to analyze the physical arrangement of blocks.

Explain one way you can see the final generalization for n *by looking at the physical arrangement of blocks. How does the block arrangement show that the final generalization is true?* (Students may explore to discover the connection; for a generalization that includes $5n$, for example, they might look for 5 groups of n blocks in the physical arrangement. The groups may be in rows at first and later in more obscure cluster arrangements that must be observed carefully.)

What do you notice about the dimensions of the arrangement? Do you see any relationship to the generalization for n?

As the designs become more complex, it can become difficult to see the generalization in the physical arrangement, and the students may rely on the constant difference method. Whatever works for the student is fine.

Reproducible "Number of Blocks" Pages

The Number of Blocks pages (pp. 55–58) are to be reproduced and distributed so that students always have the correct sheet for the section they are working on. As the students build the block designs, they should label the columns with the types of blocks required and record in the blanks the numbers of blocks used for each design. Use the chart below as a quick reference for specific information on the Number of Blocks pages.

Have students label their columns so that the answers may be easily checked (some examples are shown below). You may want them to either draw the shapes or use words to describe the blocks. One possibility, especially for young children, is to trace the actual block at the top of each answer column. Use whatever method works best for you and the students.

Number of Blocks Page	Types of Pattern Blocks	Corresponding Patterns
Section 1 — p. 55	One shape (shown in graphics on activity pages)	Patterns 1–10
Section 2 — p. 55	Two shapes (shown in graphics on activity pages)	Patterns 11–20
Section 3 — p. 55	Two shapes (shown in graphics on activity pages)	Patterns 21–30
Section 4 — pp. 56–58	Three, four, or five shapes (shown in graphics on activity pages)	Patterns 31–40
Section 5 — pp. 55–56	One, two, or three shapes (shown in graphics on activity pages)	Patterns 41–50

Design	□ Blocks	▱ Blocks	Total
1			
2			

Design	Square Blocks	Tan Rhombus Blocks	Total
1			
2			

NUMBER OF BLOCKS

PATTERN _____

Design	_____ Blocks
1	
2	
3	
4	
5	
• • •	• • •
10	
• • •	• • •
20	
• • •	• • •
n	

PATTERN _____

Design	_____ Blocks	_____ Blocks	Total
1			
2			
3			
4			
5			
• • •	• • •	• • •	• • •
10			
• • •	• • •	• • •	• • •
20			
• • •	• • •	• • •	• • •
n			

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NUMBER OF BLOCKS

PATTERN _____

Design	_____ Blocks	_____ Blocks	_____ Blocks	Total
1				
2				
3				
4				
5				
• • •	• • •	• • •	• • •	• • •
10				
• • •	• • •	• • •	• • •	• • •
20				
• • •	• • •	• • •	• • •	• • •
n				

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NUMBER OF BLOCKS

PATTERN _____

Design	_____ Blocks	_____ Blocks	_____ Blocks	_____ Blocks	Total
1					
2					
3					
4					
5					
• • •	• • •	• • •	• • •	• • •	• • •
10					
• • •	• • •	• • •	• • •	• • •	• • •
20					
• • •	• • •	• • •	• • •	• • •	• • •
n					

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NUMBER OF BLOCKS

PATTERN _____

Design	_____ Blocks	_____ Blocks	_____ Blocks	_____ Blocks	_____ Blocks	Total
1						
2						
3						
4						
5						
• • •	• • •	• • •	• • •	• • •	• • •	• • •
10						
• • •	• • •	• • •	• • •	• • •	• • •	• • •
20						
• • •	• • •	• • •	• • •	• • •	• • •	• • •
n						

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ANSWER KEY

Note: The explanations included in the answer key are intended for the teacher (for clarification on how the answer *could* be found). It is not suggested that the students be given explanations or specific directions on how to find answers; students should be allowed to discover their own unique methods for solving the problems.

Pattern 1 Answers

Design	
1	2
2	4
3	6
4	8
5	10
⋮	⋮
10	20
⋮	⋮
20	40
⋮	⋮
n	2n

A. 36
If $n = 18$, then $2n = 36$.

B. The s are always in two rows. Each row has n blocks, so the total number of blocks will be $2n$.

C. 35th
If $2n = 70$, then $n = 35$.

D. No. We always use an even number of blocks to build a design.

E. 194
If $n = 97$, then $2n = 194$.

F. 129th
If $2n = 258$, then $n = 129$.

G. The numbers grow by 2 in the column; 2 is the number in front of n in the generalization $2n$.

Pattern 2 Answers

Design	△
1	3
2	6
3	9
4	12
5	15
⋮	⋮
10	30
⋮	⋮
20	60
⋮	⋮
n	3n

A. 72
If $n = 24$, then $3n = 72$.

B. The blocks are always in a Y shape. Each arm has n blocks, so the total will be $3n$.

C. 23rd
Since $3n = 69$, n must by 23.

D. No. The number of blocks must be a multiple of 3 (86 is not).

E. 237
If $n = 79$, then $3n = 237$.

F. 86th
If $3n = 258$, then $n = 86$.

G. The numbers grow by 3; 3 is the number in front of n in the generalization $3n$.

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Pattern 3 Answers

Design	◇
1	6
2	12
3	18
4	24
5	30
⋮	⋮
10	60
⋮	⋮
20	120
⋮	⋮
n	6n

A. 102
Since $n = 17$, $6n = 102$.

B. The blocks are always arranged in the shape of a six-pointed star. For each new design, we add another outer layer to the six-pointed star. The total number of blocks will therefore be $6n$.

C. 22nd
If $6n = 132$, then $n = 22$.

D. Yes; 192 is divisible by 6 (the 32nd design will use 192 ◇s).

E. 318
If $n = 53$, then $6n = 318$.

F. 67th
Since $6n = 402$, n will be 67.

G. The numbers grow by 6 in the ◇ column; 6 is the number before n in the generalization $6n$.

Pattern 4 Answers

Design	⏢
1	4
2	8
3	12
4	16
5	20
⋮	⋮
10	40
⋮	⋮
20	80
⋮	⋮
n	4n

A. 92
If $n = 23$, then $4n = 92$.

B. The blocks are always arranged in four quadrants with n blocks in each. The total is therefore always $4n$.

C. 28th
If $4n = 112$, then $n = 28$.

D. No. We use multiples of 4 for each design (154 is not a multiple of 4).

E. 296
If $n = 74$, then $4n = 296$.

F. 89th
If $4n = 356$, then $n = 89$.

G. The numbers grow by 4 in the ⏢ column; 4 is the number in front of n in the generalization $4n$.

Pattern 5 Answers

Design	△
1	5
2	10
3	15
4	20
5	25
⋮	⋮
10	50
⋮	⋮
20	100
⋮	⋮
n	5n

A. 135
If $n = 27$, then $5n = 135$.

B. The blocks are always arranged as a five-pointed star with n blocks on each line of the star. This gives a total of $5n$ blocks.

C. 31st
If $5n = 155$, then $n = 31$.

D. Yes; 350 is divisible by 5. (The 70th design will use 350 blocks.)

E. 435
If $n = 87$, then $5n = 435$.

F. 96th
If $5n = 480$, then $n = 96$.

G. The numbers grow by 5 in the △ column; 5 is the number before n in the generalization $5n$.

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Pattern 6 Answers

Design	⬡
1	3
2	5
3	7
4	9
5	11
⋮	⋮
10	21
⋮	⋮
20	41
⋮	⋮
n	$2n + 1$

A. 51
If $n = 25$, then $2n + 1 = 51$.

B. The blocks are always arranged in two rows; there are n blocks in the top row and $n + 1$ blocks in the bottom row. This gives a total of $n + (n + 1) = 2n + 1$.

C. 69th
If $2n + 1 = 139$, then $n = 69$.

D. The block arrangements are almost the same. The only difference is the extra block at the left side in the bottom row on Pattern 6.

E. 159
If $n = 79$, then $2n + 1 = 159$.

F. 198th
If $2n + 1 = 397$, then $n = 198$.

G. The numbers grow by 2; 2 appears before n in the generalization $2n + 1$.

Pattern 7 Answers

Design	◇
1	3
2	7
3	11
4	15
5	19
⋮	⋮
10	39
⋮	⋮
20	79
⋮	⋮
n	$4n - 1$

A. 115
If $n = 29$, then $4n - 1 = 115$.

B. The blocks are arranged in a V shape inside a wider V. Three arms have n blocks, one arm has $n - 1$ blocks; the total will be $3n + (n - 1) = 4n - 1$.

C. 35th
If $4n - 1 = 139$, then $n = 35$.

D. No. There is no whole number n that satisfies the statement $4n - 1 = 161$.

E. 19
Each center V has $2n - 1$ blocks.

F. 351
If $n = 88$, then $4n - 1 = 351$.

G. 93rd
If $4n - 1 = 371$, then $n = 93$.

H. The numbers grow by 4; 4 is the number before n in the generalization $4n - 1$.

Pattern 8 Answers

Design	⬡
1	5
2	8
3	11
4	14
5	17
⋮	⋮
10	32
⋮	⋮
20	62
⋮	⋮
n	$3n + 2$

A. 83
If $n = 27$, then $3n + 2 = 83$.

B. The arrangement of the blocks resembles 3 rows (divide the extra 2 blocks at either end between the top and bottom rows). There are always $n + 1$ blocks in both the top and bottom rows and n blocks in the middle row. This gives a total of $2(n + 1) + n = 2n + 2 + n = 3n + 2$ blocks.

C. 43rd
If $3n + 2 = 131$, then $n = 43$.

D. An odd number. Even-numbered designs use an even number of blocks, odd-numbered designs use an odd number of blocks. Design 79 uses 239 blocks.

E. 58, 156
The number of outside blocks is always $2n + 2$.

F. 260
If $n = 86$, then $3n + 2 = 260$.

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G. 95th
If $3n + 2 = 287$, then $n = 95$.
H. The numbers grow by 3; 3 is the number before n in the generalization $3n + 2$.

Pattern 9 Answers

Design	△
1	4
2	7
3	10
4	13
5	16
⋮	⋮
10	31
⋮	⋮
20	61
⋮	⋮
n	3n + 1

A. 88
If $n = 29$, then $3n + 1 = 88$.
B. The blocks are always in the shape of an upside-down Y with n blocks on each arm and one constant block in the middle. This gives a total of $3n + 1$ blocks.
C. 49th
If $3n + 1 = 148$, then $n = 49$.
D. An odd number. Even number designs always use an odd number of blocks (three times an even number gives another even; then we add the constant 1).
E. The pattern pictures are the same except for a constant block in the middle of Pattern 9. Therefore, the answers are $3n$ (Pattern 2) and $3n + 1$ (Pattern 9).
F. 280
If $n = 93$, then $3n + 1 = 280$.
G. 84th
If $3n + 1 = 253$, then $n = 84$.
H. The numbers grow by 3; 3 is the number in front of n in the generalization $3n + 1$.

Pattern 10 Answers

Design	◇
1	5
2	9
3	13
4	17
5	21
⋮	⋮
10	41
⋮	⋮
20	81
⋮	⋮
n	4n + 1

A. 109
If $n = 27$, then $4n + 1 = 109$.
B. The arrangement of blocks is similar to an X, with n blocks on each arm and one constant block in the middle. The total is always $4n + 1$.
C. 37th
If $4n + 1 = 149$, then $n = 37$.
D. No. There is no whole number n that satisfies the statement $4n + 1 = 179$.
E. The numbers grow by 4; 4 is the number before n in the answer $4n + 1$.
F. 393
If $n = 98$, then $4n + 1 = 393$.
G. 113th
If $4n + 1 = 453$, then $n = 113$.
H. One possible answer follows:

Pattern 11 Answers

Design	□	△	Total
1	1	1	2
2	2	2	4
3	3	3	6
4	4	4	8
5	5	5	10
⋮	⋮	⋮	⋮
10	10	10	20
⋮	⋮	⋮	⋮
20	20	20	40
⋮	⋮	⋮	⋮
n	n	n	2n

A. 31, 31
For design 31, □ = n, △ = n, and $n = 31$.
B. □: The numbers are the counting numbers.
Total: The numbers are all the even numbers (the multiples of 2).
C. 43rd
If $2n = 86$, then $n = 43$.
D. The blocks are always in two rows with n blocks in each row. The total will always be $2n$.
E. 79, 79

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For the 79th design, □ = n, △ = n, and n = 79.

F. 146th
If $2n$ = 292, then n = 146.

G. The numbers grow by 2; 2 is the number before n in the generalization $2n$.

Pattern 12 Answers

Design	⬡	△	Total
1	1	2	3
2	2	3	5
3	3	4	7
4	4	5	9
5	5	6	11
⋮	⋮	⋮	⋮
10	10	11	21
⋮	⋮	⋮	⋮
20	20	21	41
⋮	⋮	⋮	⋮
n	n	n + 1	2n + 1

A. 34, 35
For the 34th design, n = 34 and $n + 1$ = 35.

B. ⬡: The numbers are consecutive (counting) numbers, starting with 1.
△: The numbers are consecutive (counting) numbers, starting with 2.
Total: The numbers increase by 2, starting with 3 (the odd numbers).

C. 48th
If $2n + 1$ = 97, then n = 48.

D. There are always n ⬡s in each row and $n + 1$ △s in each row for a total of $n + (n + 1) = 2n + 1$ blocks.

E. 87, 88
For design 87, n = 87 and $n + 1$ = 88.

F. 151st
If $2n + 1$ = 303, then n = 151.

G. The numbers grow by 2; 2 is the number in front of n in the generalization $2n + 1$.

Pattern 13 Answers

Design	△	□	Total
1	1	0	1
2	2	1	3
3	3	2	5
4	4	3	7
5	5	4	9
⋮	⋮	⋮	⋮
10	10	9	19
⋮	⋮	⋮	⋮
20	20	19	39
⋮	⋮	⋮	⋮
n	n	n - 1	2n - 1

A. 39, 38
For design 39, n = 39 and $n - 1$ = 38.

B. 47th
If $2n - 1$ = 93, then n = 47.

C. There are always $n - 1$ □s in the middle and n △s around the outside for a total of $(n - 1) + n = 2n - 1$ blocks.

D. The numbers grow by 2; 2 is before n in our generalization $2n - 1$.

E. 187
If $n - 1$ = 93, then n = 94 and $2n - 1$ = 187.

F. 109, 108
If $2n - 1$ = 217, then n = 109 and $n - 1$ = 108.

G. When we add n for the △ column with $n - 1$ for the □ column, we have two n's, or $2n$, and also –1; our total must be $2n - 1$.

Pattern 14 Answers

Design	△	◇	Total
1	1	2	3
2	2	4	6
3	3	6	9
4	4	8	12
5	5	10	15
⋮	⋮	⋮	⋮
10	10	20	30
⋮	⋮	⋮	⋮
20	20	40	60
⋮	⋮	⋮	⋮
n	n	2n	3n

A. 37, 74
For design 37, n = 37 and $2n$ = 74.

B. △: The numbers increase by 1 and are the counting numbers.
◇: The numbers increase by 2 and are the even numbers.
Total: The numbers increase by 3 and are the multiples of 3.

C. 43rd
If $3n$ = 129, then n = 43.

D. There are always n "houses" in each row and each house uses 3 blocks, so the total will always be $3n$ blocks.

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E. △: The numbers increase by 1.
<>: The numbers increase by 2.
Total: The numbers increase by 3.
These are the numbers before n in our generalizations for the columns: n, $2n$, and $3n$.

F. When we add one n and two n's, we have three n's, or $3n$.

Pattern 15 Answers

Design	▱	△	Total
1	1	4	5
2	3	8	11
3	5	12	17
4	7	16	23
5	9	20	29
⋮	⋮	⋮	⋮
10	19	40	59
⋮	⋮	⋮	⋮
20	39	80	119
⋮	⋮	⋮	⋮
n	2n − 1	4n	6n − 1

A. 69, 140
If $n = 35$, then $2n - 1 = 69$ and $4n = 140$.

B. The ▱s are in a V shape in which the two arms have n and $n - 1$ blocks. The △s are in a double V shape with n blocks on each arm (the top arms use the end blocks). Therefore, the total number of blocks is always $4n + n + (n - 1)$, or $6n - 1$.

C. 47th
If $6n - 1 = 281$, then $n = 47$.

D. 319 ▱s: Yes. The 160th design uses 319 ▱s.
319 blocks: No. There is no whole number n that satisfies the equation $6n - 1 = 319$.

E. ▱: The numbers grow by 2.
△: The numbers grow by 4.
Total: The numbers grow by 6.
The constant differences are in front of n in the generalizations $2n - 1$, $4n$, and $6n - 1$.

F. When we add $2n$ –1 and $4n$, we have six n's, or $6n$, and we still have –1, so we have $(2n - 1) + 4n = 6n - 1$.

Pattern 16 Answers

Design	<>	△	Total
1	3	6	9
2	4	8	12
3	5	10	15
4	6	12	18
5	7	14	21
⋮	⋮	⋮	⋮
10	12	24	36
⋮	⋮	⋮	⋮
20	22	44	66
⋮	⋮	⋮	⋮
n	n + 2	2n + 4	3n + 6

A. 40, 80
If $n = 38$, then $n + 2 = 40$ and $2n + 4 = 80$.

B. 88
If $n + 2 = 44$, then $n = 42$ and $2n + 4 = 88$.

C. The designs are always arranged as rings of 3-block clusters. There are $n + 2$ of these clusters for a total of $3(n + 2) = 3n + 6$ blocks.

D. <>: The numbers grow by 1.
△: The numbers grow by 2.
Total: The numbers grow by 3.
The constant differences are the numbers in front of n in our generalizations $n + 2$, $2n + 4$, and $3n + 6$.

E. 53, 159
If $2n + 4 = 106$, then $n = 51$, $n + 2 = 53$, and $3n + 6 = 159$.

F. 87
If $3n + 6 = 261$, then $n = 85$ and $n + 2 = 87$.

G. When we add $n + 2$ and $2n + 4$, we have three n's, or $3n$, and we have $2 + 4 = 6$. The total is therefore $(n + 2) + (2n + 4) = 3n + 6$.

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Pattern 17 Answers

Design	▱	△	Total
1	1	0	1
2	2	2	4
3	3	4	7
4	4	6	10
5	5	8	13
⋮	⋮	⋮	⋮
10	10	18	28
⋮	⋮	⋮	⋮
20	20	38	58
⋮	⋮	⋮	⋮
n	n	2n – 2	3n – 2

A. 37, 72
For design 37, $n = 37$ and $2n - 2 = 72$.

B. The blocks are always in three rows with $n - 1$ △s each in the top and bottom rows and n ▱s in the middle row. This gives a total of $(n - 1) + (n - 1) + n = 3n - 2$ blocks.

C. 140 ▱s: Yes. The 140th design will use 140 ▱s.
140 △s: Yes. The 71st design will use 140 △s.
140 blocks: No. There is no whole number n that satisfies the equation $3n - 2 = 140$.

D. ▱: The numbers grow by 1.
△: The numbers grow by 2.
Total: The numbers grow by 3.
The constant differences are the numbers in front of n in the generalizations n, $2n - 2$, and $3n - 2$.

E. 56, 166
If $2n - 2 = 110$, then $n = 56$ and $3n - 2 = 166$.

F. 69
If $3n - 2 = 205$, then $n = 69$.

G. 186
If $3n - 2 = 280$, then $n = 94$ and $2n - 2 = 186$.

H. When we add n and $2n - 2$, we get three n's, or $3n$, and -2, so the sum is $3n - 2$.

Pattern 18 Answers

Design	□	▱	Total
1	2	3	5
2	3	5	8
3	4	7	11
4	5	9	14
5	6	11	17
⋮	⋮	⋮	⋮
10	11	21	32
⋮	⋮	⋮	⋮
20	21	41	62
⋮	⋮	⋮	⋮
n	n + 1	2n + 1	3n + 2

A. 40, 79
Since $n = 39$, $n + 1 = 40$ and $2n + 1 = 79$.

B. 125
If $n + 1 = 63$, then $n = 62$ and $2n + 1 = 125$.

C. There are always $n + 1$ each of the ▱ and □ blocks around the outside as well as n ▱s in the center of each design. This gives a total of $2(n + 1) + n$ or $3n + 2$ blocks.

D. □: The numbers grow by 1.
▱: The numbers grow by 2.
Total: The numbers grow by 3.
The constant differences are the numbers in front of n in our generalizations $n + 1$, $2n + 1$, and $3n + 2$.

E. 70, 209
If $2n + 1 = 139$, then $n = 69$, $n + 1 = 70$, and $3n + 2 = 209$.

F. 163
If $3n + 2 = 245$, then $n = 81$ and $2n + 1 = 163$.

G. When we add $n + 1$ and $2n + 1$, we get three n's, or $3n$, and $1 + 1 = 2$ for a total of $3n + 2$.

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Pattern 19 Answers

Design	▱	△	Total
1	1	4	5
2	2	6	8
3	3	8	11
4	4	10	14
5	5	12	17
⋮	⋮	⋮	⋮
10	10	22	32
⋮	⋮	⋮	⋮
20	20	42	62
⋮	⋮	⋮	⋮
n	n	2n + 2	3n + 2

A. 35, 72
For design 35, $n = 35$ and $2n + 2 = 72$.

B. 118
If $n = 58$, then $2n + 2 = 118$.

C. There are always n ▱ blocks across the middle of each design and $n + 1$ △s across the top and across the bottom of each design. This gives a total of $2(n + 1) + n$ or $3n + 2$ blocks in each design.

D. ▱: The numbers grow by 1.
△: The numbers grow by 2.
Total: The numbers grow by 3.
The constant differences are the numbers in front of n in our generalizations for each column: n, $2n + 2$, and $3n + 2$.

E. 62, 188
If $2n + 2 = 126$, then $n = 62$ and $3n + 2 = 188$.

F. 67
If $3n + 2 = 203$, then $n = 67$.

G. 160
If $3n + 2 = 239$, then $n = 79$ and $2n + 2 = 160$.

H. When we add n and $2n + 2$, we have three n's, or $3n$, and 2 for a total of $3n + 2$.

Pattern 20 Answers

Design	△	◇	Total
1	2	8	10
2	3	11	14
3	4	14	18
4	5	17	22
5	6	20	26
⋮	⋮	⋮	⋮
10	11	35	46
⋮	⋮	⋮	⋮
20	21	65	86
⋮	⋮	⋮	⋮
n	n + 1	3n + 5	4n + 6

A. 39, 119
When $n = 38$, $n + 1 = 39$ and $3n + 5 = 119$.

B. 203
When $n + 1 = 67$, $n = 66$ and $3n + 5 = 203$.

C. Each design has $n + 1$ clusters of blocks spaced around the constant 2 blocks in the middle. Each cluster contains 4 blocks, so the total is $4(n + 1) + 2 = 4n + 4 + 2 = 4n + 6$ blocks in each design.

D. △: The numbers grow by 1.
◇: The numbers grow by 3.
Total: The numbers grow by 4.
The constant differences are the numbers in front of n in our generalizations $n + 1$, $3n + 5$, and $4n + 6$.

E. 44, 178
When $3n + 5 = 134$, $n = 43$, $n + 1 = 44$, and $4n + 6 = 178$.

F. 68, 206
If $4n + 6 = 274$, then $n = 67$, $n + 1 = 68$, and $3n + 5 = 206$.

G. No. The number will never be odd because $4n$ will always be even for any whole number n, and then we add 6, which will always give us another even number.

H. When we add $n + 1$ and $3n + 5$, we get four n's, or $4n$, and $1 + 5 = 6$ for a total of $4n + 6$.

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Pattern 21 Answers

Design	⬡	▱	Total
1	1	0	1
2	2	2	4
3	3	4	7
4	4	6	10
5	5	8	13
⋮	⋮	⋮	⋮
10	10	18	28
⋮	⋮	⋮	⋮
20	20	38	58
⋮	⋮	⋮	⋮
n	n	2n – 2 or 2(n – 1)	3n – 2

A. 36, 70
For design 36, $n = 36$ and $2n - 2 = 70$.

B. 92
Since $n = 47$, $2n - 2 = 92$.

C. 41
If $2(n - 1) = 80$, then $n = 41$.

D. There are always n ⬡s in the middle row. There are also $n - 1$ ▱s as a top row and again as a bottom row. This gives a total of $n + 2(n - 1) = 3n - 2$ blocks in each design.

E. ⬡: The numbers grow by 1.
▱: The numbers grow by 2.
Total: The numbers grow by 3.
The constant differences are the numbers in front of n in our generalizations n, $2n - 2$, and $3n - 2$.

F. No; $2n$ will be an even number; when you subtract 2 you still have an even number.

G. 93, 184
If $3n - 2 = 277$, then $n = 93$ and $2n - 2 = 184$.

H. When we add $n + (2n - 2)$ we have three n's, or $3n$, and we still have –2 for a total of $3n - 2$.

Pattern 22 Answers

Design	▱	◇	Total
1	1	1	2
2	3	3	6
3	5	5	10
4	7	7	14
5	9	9	18
⋮	⋮	⋮	⋮
10	19	19	38
⋮	⋮	⋮	⋮
20	39	39	78
⋮	⋮	⋮	⋮
n	2n – 1	2n – 1	4n – 2

A. 77, 77
If $n = 39$, then $2n - 1 = 77$ for both ▱ and ◇.

B. 59, 118
Since $2n - 1 = 59$, $n = 30$ and $4n - 2 = 118$.

C. In each design, for both ▱s and ◇s, there are always $2n - 1$ of each type of block for a total of $2(2n - 1) = 4n - 2$.

D. ▱: The numbers grow by 2.
◇: The numbers grow by 2.
Total: The numbers grow by 4.
These constant differences are the numbers in front of n in our generalizations $2n - 1$, $2n - 1$, and $4n - 2$.

E. No; $4n$ will always be even; subtracting 2 (another even number) makes $4n - 2$ even.

F. 137, 137
If $4n - 2 = 274$, then $n = 69$ and $2n - 1 = 137$.

G. As the design number increases, the center of our design more closely approaches the shape of a circle.

H. When we add $2n - 1$ and $2n - 1$, we have four n's, or $4n$, and $(-1) + (-1) = -2$ for a total of $4n - 2$.

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Pattern 23 Answers

Design	⬡	□	Total
1	1	6	7
2	7	12	19
3	13	18	31
4	19	24	43
5	25	30	55
⋮	⋮	⋮	⋮
10	55	60	115
⋮	⋮	⋮	⋮
20	115	120	235
⋮	⋮	⋮	⋮
n	6n – 5	6n	12n – 5

A. 205, 210
If $n = 35$, then $6n - 5 = 205$ and $6n = 210$.

B. 264, 523
If $6n - 5 = 259$, then $6n = 264$, $n = 44$, and $12n - 5 = 523$.

C. The patterns begin with the number 6, and each number is 6 greater than the number before. These are the multiples of 6.

D. No. For all values of n, $6n$ will be an even number. When we subtract 5, we will always have an odd number.

E. ⬡: The numbers grow by 6.
□: The numbers grow by 6.
Total: The numbers grow by 12.
The constant differences are the numbers in front of n in our generalizations $6n - 5$, $6n$, and $12n - 5$.

F. 337, 342
If $12n - 5 = 679$, then $n = 57$, $6n - 5 = 337$, and $6n = 342$.

G. Each design has six arms with $2n - 1$ blocks in each arm. There is also the one ⬡ in the center, so the total is $6(2n - 1) + 1 = (12n - 6) + 1 = 12n - 5$.

H. When we add $6n - 5$ and $6n$, we have twelve n's, or $12n$, and -5 for a total of $12n - 5$.

Pattern 24 Answers

Design	⬡	△	Total
1	1	6	7
2	2	10	12
3	3	14	17
4	4	18	22
5	5	22	27
⋮	⋮	⋮	⋮
10	10	42	52
⋮	⋮	⋮	⋮
20	20	82	102
⋮	⋮	⋮	⋮
n	n	4n + 2	5n + 2

A. 38, 154
For design 38, $n = 38$ and $4n + 2 = 154$.

B. 45, 227
If $4n + 2 = 182$, then $n = 45$ and $5n + 2 = 227$.

C. 272 ⬡s: Yes. The 272nd design will use 272 ⬡s.
272 △s: No. There is no whole number n that satisfies $4n + 2 = 272$.
272 blocks: Yes. The 54th design will use 272 blocks.

D. No. The number of triangles will always be $4n + 2$; $4n$ will always be an even number and when we add 2, the result will be even.

E. ⬡: The numbers grow by 1.
△: The numbers grow by 4.
Total: The numbers grow by 5.
The constant differences are the numbers in front of n in our generalizations n, $4n + 2$, and $5n + 2$.

F. Each design has n ⬡s and four rows of △s. The top and bottom rows of △s have n blocks and the middle two rows have $n + 1$ blocks each. This will give a total of $n + 2n + 2(n + 1) = 5n + 2$.

G. 83, 334
If $5n + 2 = 417$, then $n = 83$ and $4n + 2 = 334$.

H. When we add n and $4n + 2$, we have five n's, or $5n$, and 2 for a total of $5n + 2$.

Pattern 25 Answers

A. 100, 102
If $n = 34$, then $3n - 2 = 100$ and $3n = 102$.

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Design	△	⏢	Total
1	1	3	4
2	4	6	10
3	7	9	16
4	10	12	22
5	13	15	28
⋮	⋮	⋮	⋮
10	28	30	58
⋮	⋮	⋮	⋮
20	58	60	118
⋮	⋮	⋮	⋮
n	3n – 2	3n	6n – 2

B. 144, 286
If $3n - 2 = 142$, then $3n = 144$, $n = 48$, and $6n - 2 = 286$.

C. 316 △s: Yes. The 106th design will use 316 △s.
316 ⏢s: No. There is no whole number n that satisfies $3n = 316$.
316 blocks: Yes. The 53rd design will use 316 blocks.

D. Each design has three arms and one center block, which is a △.
Each arm has $2n - 1$ blocks, for a subtotal of $6n - 3$. Add the center block for a total of $6n - 2$.

E. △s: The numbers grow by 3.
⏢: The numbers grow by 3.
Total: The numbers grow by 6.
These are the numbers in front of n in our generalizations $3n - 2$, $3n$, and $6n - 2$.

F. 211, 213
If $6n - 2 = 424$, then $n = 71$, $3n - 2 = 211$, and $3n = 213$.

G. When we add $3n - 2$ and $3n$, we have six n's, or $6n$, and -2 for a total of $6n - 2$.

Pattern 26 Answers

Design	△	◇	Total
1	3	3	6
2	6	9	15
3	9	15	24
4	12	21	33
5	15	27	42
⋮	⋮	⋮	⋮
10	30	57	87
⋮	⋮	⋮	⋮
20	60	117	177
⋮	⋮	⋮	⋮
n	3n	6n – 3	9n – 3

A. 114, 225
If $n = 38$, then $3n = 114$ and $6n - 3 = 225$.

B. 93, 141
If $3n = 48$, then $n = 16$, $6n - 3 = 93$, and $9n - 3 = 141$.

C. 243 △s: Yes. The 81st design will use 243 △s.
243 ◇: Yes. The 41st design will use 243 ◇s.
243 blocks: No. No whole number n satisfies $9n - 3 = 243$.

D. Every design has three arms, each arm has $3n$ blocks, for a total of $3(3n) = 9n$ blocks; but when you count the blocks, three are always counted twice, so you have to subtract 3 from the total: $9n - 3$.

E. △: The numbers grow by 3.
◇: The numbers grow by 6.
Total: The numbers grow by 9.
These are the numbers in front of n in our generalizations $3n$, $6n - 3$, and $9n - 3$.

F. 171, 339
If $9n - 3 = 510$, then $n = 57$, $3n = 171$, and $6n - 3 = 339$.

G. When we add $3n$ and $6n - 3$, we have nine n's, or $9n$ and -3 for a total of $9n - 3$.

Pattern 27 Answers

Design	□	◇	Total
1	1	4	5
2	5	8	13
3	9	12	21
4	13	16	29
5	17	20	37
⋮	⋮	⋮	⋮
10	37	40	77
⋮	⋮	⋮	⋮
20	77	80	157
⋮	⋮	⋮	⋮
n	4n – 3	4n	8n – 3

A. 141, 144
If $n = 36$, then $4n - 3 = 141$ and $4n = 144$.

B. 204, 405
If $4n - 3 = 201$, then $n = 51$, $4n = 204$, and $8n - 3 = 405$.

C. 501 □s: Yes. The 126th design will use 501 □s.
501 ◇: No. There is no whole number n that satisfies $4n = 501$.
501 blocks: Yes. The 63rd design will use 501 blocks.

D. Each design always has one □ in the middle and four arms with $2n - 1$ blocks each; therefore, the total number of blocks will be $4(2n - 1) + 1$, or $8n - 4 + 1 = 8n - 3$.

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E. □: The numbers grow by 4.
<>: The numbers grow by 4.
Total: The numbers grow by 8.
These are the numbers in front of n in our generalizations $4n - 3$, $4n$, and $8n - 3$.

F. 345, 348
If $8n - 3 = 693$, then $n = 87$, $4n - 3 = 345$, and $4n = 348$.

G. When we add $4n - 3$ and $4n$, we have eight n's, or $8n$, and -3 for a total of $8n - 3$.

Pattern 28 Answers

Design	△	⏢	Total
1	4	9	13
2	7	15	22
3	10	21	31
4	13	27	40
5	16	33	49
⋮	⋮	⋮	⋮
10	31	63	94
⋮	⋮	⋮	⋮
20	61	123	184
⋮	⋮	⋮	⋮
n	3n + 1	6n + 3	9n + 4

A. 115, 231
If $n = 38$, then $3n + 1 = 115$ and $6n + 3 = 231$.

B. 285, 427
If $3n + 1 = 142$, then $n = 47$, $6n + 3 = 285$, and $9n + 4 = 427$.

C. 463 △s: Yes. The 154th design will use 463 △s.
463 ⏢s: No. No whole number n satisfies $6n + 3 = 463$.
463 blocks: Yes. The 51st design will use 463 blocks.

D. There is always one △ in the center of each design. Each design has three arms with $3n + 1$ blocks each. This gives a total of $3(3n + 1) + 1 = 9n + 3 + 1 = 9n + 4$.

E. △: The numbers grow by 3.
⏢: The numbers grow by 6.
Total: The numbers grow by 9.
These are the numbers in front of n in our generalizations $3n + 1$, $6n + 3$, and $9n + 4$.

F. 190, 381
If $9n + 4 = 571$, then $n = 63$, $3n + 1 = 190$, and $6n + 3 = 381$.

G. When we add $3n + 1$ and $6n + 3$, we have nine n's, or $9n$, and $1 + 3$, or 4, for a total of $9n + 4$.

Pattern 29 Answers

Design	▱	◇	Total
1	1	4	5
2	5	16	21
3	9	28	37
4	13	40	53
5	17	52	69
⋮	⋮	⋮	⋮
10	37	112	149
⋮	⋮	⋮	⋮
20	77	232	309
⋮	⋮	⋮	⋮
n	4n - 3	12n - 8	16n - 11

A. 137, 412
If $n = 35$, then $4n - 3 = 137$ and $12n - 8 = 412$.

B. 544, 725
If $4n - 3 = 181$, then $n = 46$, $12n - 8 = 544$, and $16n - 11 = 725$.

C. There is always one ▱ in the center of each design. There are four arms, each with $n - 1$ ▱s and $3n - 2$ <>s for a grand total of $4(n - 1) + 4(3n - 2)$ blocks. This equals $4n - 4 + 12n - 8 + 1$ or $16n - 11$ blocks.

D. ▱: The numbers grow by 4.
<>: The numbers grow by 12.
Total: The numbers grow by 16.
These are the numbers in front of n in our generalizations $4n - 3$, $12n - 8$, and $16n - 11$.

E. 189, 568
If $16n - 11 = 757$, then $n = 48$, $4n - 3 = 189$, and $12n - 8 = 568$.

F. When we add $4n - 3$ and $12n - 8$, we have sixteen n's, or $16n$, and $(-3) + (-8)$, or -11; thus, the final answer is $16n - 11$.

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Pattern 30 Answers

Design	△	◇	Total
1	10	17	27
2	17	24	41
3	24	31	55
4	31	38	69
5	38	45	83
⋮	⋮	⋮	⋮
10	73	80	153
⋮	⋮	⋮	⋮
29	143	150	293
⋮	⋮	⋮	⋮
n	7n + 3	7n +10	14n + 13

A. 276, 283
When $n = 39$, $7n + 3 = 276$ and $7n + 10 = 283$.

B. 325, 657
If $7n + 10 = 332$, then $n = 46$, $7n + 3 = 325$, and $14n + 13 = 657$.

C. There is always a center core of 3 △s and 3 ◇s for each design. There are 14 arms extending away from the center: 7 arms of n blocks and 7 arms of $n + 1$ blocks, alternating. The total is therefore $7n + 7(n + 1) + 3 + 3 = 7n + 7n + 7 + 3 + 3 = 14n + 7 + 3 + 3 = 14n + 13$.

D. △: The numbers grow by 7.
◇: The numbers grow by 7.
Total: The numbers grow by 14.
These are the numbers in front of n in our generalizations $7n + 3$, $7n + 10$, and $14n + 13$.

E. 381, 755
If $7n + 3 = 374$, then $n = 53$, $7n + 10 = 381$, and $14n + 13 = 755$.

F. 472, 479
If $14n + 13 = 951$, then $n = 67$, $7n + 3 = 472$, and $7n + 10 = 479$.

G. When we add $7n + 3$ and $7n + 10$, we have fourteen n's, or $14n$, and 3 + 10, or 13, for a final total of $14n + 13$.

Pattern 31 Answers

Design	⏢	◇	□	Total
1	1	2	1	4
2	2	4	2	8
3	3	6	3	12
4	4	8	4	16
5	5	10	5	20
⋮	⋮	⋮	⋮	⋮
10	10	20	10	40
⋮	⋮	⋮	⋮	⋮
20	20	40	20	80
⋮	⋮	⋮	⋮	⋮
n	n	2n	n	4n

A. 35, 70, 35
For design 35, $n = 35$ and $2n = 70$.

B. 44, 44, 176
If $2n = 88$, then $n = 44$ and $4n = 176$.

C. Each design always has n "houses" and each house is made of 4 blocks, so the total is $4n$ blocks.

D. ◇: The numbers increase by 2s (the even numbers or multiples of 2).
Total: The numbers increase by 4s (the multiples of 4).

E. 67, 134, 67
If $4n = 268$, then $n = 67$ and $2n = 134$.

F. When we combine n, $2n$, and n, we have four n's or $4n$.

Pattern 32 Answers

Design	△	▱	◇	Total
1	2	2	2	6
2	3	3	3	9
3	4	4	4	12
4	5	5	5	15
5	6	6	6	18
⋮	⋮	⋮	⋮	⋮
10	11	11	11	33
⋮	⋮	⋮	⋮	⋮
20	21	21	21	63
⋮	⋮	⋮	⋮	⋮
n	n + 1	n + 1	n + 1	3 (n + 1) or 3n + 3

A. 39, 39, 39
If $n = 38$, then $n + 1 = 39$.

B. 138
If $n + 1 = 46$, then $n = 45$ and $3n + 3 = 138$.

C. Every design is made up of three rings, each containing $n + 1$ blocks. The total is therefore $3(n + 1)$ blocks.

D. 83, 249
When we use 83 ◇s, $n + 1 = 83$, $n = 82$, and $3n + 3 = 249$.

E. No. The total number must be a multiple of 3; the number 200 is not a multiple of 3. No whole number n satisfies $3n + 3 = 200$.

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F. 91, 91, 91
If $3n + 3 = 273$, then $n = 90$ and $n + 1 = 91$.

G. When we combine $n + 1$, $n + 1$, and $n + 1$ we get three n's, or $3n$, and $1 + 1 + 1 = 3$. The total is $3n + 3$.

Pattern 33 Answers

Design	⬡	□	◇	Total
1	1	2	4	7
2	2	4	8	14
3	3	6	12	21
4	4	8	16	28
5	5	10	20	35
⋮	⋮	⋮	⋮	⋮
10	10	20	40	70
⋮	⋮	⋮	⋮	⋮
20	20	40	80	120
⋮	⋮	⋮	⋮	⋮
n	n	2n	4n	7n

A. 37, 74, 148
For design 37, $n = 37$, $2n = 74$, and $4n = 148$.

B. 32, 128, 224
Since $2n = 64$, $n = 32$, $4n = 128$, and $7n = 224$.

C. Each design is made up of n "octagons," with 7 blocks each, so the total must be $7n$ blocks.

D. ⬡: The numbers grow by 1.
□: The numbers grow by 2.
◇: The numbers grow by 4
Total: The numbers grow by 7.
These are the numbers in front of n in our generalizations.

E. 61, 122, 244
If $7n = 427$, then $n = 61$, $2n = 122$, and $4n = 244$.

F. When we combine n, $2n$, and $4n$, we have seven n's, or $7n$, for the total.

Pattern 34 Answers

Design	△	□	◇	Total
1	1	3	3	7
2	4	6	9	19
3	7	9	15	31
4	10	12	21	43
5	13	15	27	55
⋮	⋮	⋮	⋮	⋮
10	28	30	57	115
⋮	⋮	⋮	⋮	⋮
20	58	60	117	235
⋮	⋮	⋮	⋮	⋮
n	3n – 2	3n	6n – 3	12n – 5

A. 112, 114, 225
If $n = 38$, then $3n - 2 = 112$, $3n = 114$, and $6n - 3 = 225$.

B. 85, 87, 343
If $6n - 3 = 171$, then $n = 29$, $3n - 2 = 85$, $3n = 87$, and $12n - 5 = 343$.

C. 243 △s: No. There is no whole number n that will satisfy the equation $3n - 2 = 243$.
243 □s: Yes. The 81st design will use 243 □s.
243 ◇s: Yes. The 41st design will use 243 ◇s.
243 blocks: No. There is no whole number n that will satisfy the equation $12n - 5 = 243$.

D. There is always a center core of 7 blocks in each design. Then, there are 3 arms with $4(n - 1)$ blocks each. The total is $7 + 3[4(n - 1)] = 7 + 3(4n - 4) = 7 + 12n - 12 = 12n - 5$.

E. △: The numbers grow by 3.
□: The numbers grow by 3.
◇: The numbers grow by 6.
Total: The numbers grow by 12.
These are the numbers in front of n in the generalizations.

F. 139, 141, 279
If $12n - 5 = 559$, then $n = 47$, $3n - 2 = 139$, $3n = 141$, and $6n - 3 = 279$.

G. When we combine $3n - 2$, $3n$, and $6n - 3$, we have twelve n's, or $12n$, and $(-2) + (-3) = -5$ for a total of $12n - 5$.

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Pattern 35 Answers

Design	▱	△	⬡	Total
1	3	3	0	6
2	6	9	1	16
3	9	15	2	26
4	12	21	3	36
5	15	27	4	46
⋮	⋮	⋮	⋮	⋮
10	30	57	9	96
⋮	⋮	⋮	⋮	⋮
20	60	117	19	116
⋮	⋮	⋮	⋮	⋮
n	3n	6n – 3	n – 1	10n – 4

A. 105, 207, 34
Since $n = 35$, $3n = 105$, $6n - 3 = 207$, and $n - 1 = 34$.

B. 126, 41, 416
If $6n - 3 = 249$, then $n = 42$, $3n = 126$, $n - 1 = 41$, and $10n - 4 = 416$.

C. 279 ▱s: Yes. The 93rd design will use 279 ▱s.
279 △s: Yes. The 47th design will use 279 △s.
279 ⬡ s: Yes. The 280th design will use 279 ⬡ s.
279 blocks: No. No whole number n satisfies the equation $10n - 4 = 279$.

D. Each design has n of the basic triangles of 6 blocks (shown as the first design). There are also $n - 1$ of the "in-between" triangles, which have 4 blocks each. The total will be $6n + 4(n - 1)$, or $6n + 4n - 4 = 10n - 4$ blocks.

E. ▱: The numbers grow by 3.
△: The numbers grow by 6.
⬡: The numbers grow by 1.
Total: The numbers grow by 10.
These are the numbers in front of n in the generalizations.

F. 153, 303, 50
If $10n - 4 = 506$, then $n = 51$, $3n = 153$, $6n - 3 = 303$, and $n - 1 = 50$.

G. When we combine $3n$, $6n - 3$, and $n - 1$, we have ten n's, or $10n$, and $(-3) + (-1) = -4$ for a total of $10n - 4$.

Pattern 36 Answers

Design	▱	□	◇	Total
1	1	4	4	9
2	5	8	12	25
3	9	12	20	41
4	13	16	28	57
5	17	20	36	73
⋮	⋮	⋮	⋮	⋮
10	37	40	76	153
⋮	⋮	⋮	⋮	⋮
20	77	80	156	313
⋮	⋮	⋮	⋮	⋮
n	4n – 3	4n	8n – 4	16n – 7

A. 149, 152, 300
If $n = 38$, then $4n - 3 = 149$, $4n = 152$, and $8n - 4 = 300$.

B. 177, 356, 713
If $4n = 180$, then $n = 45$, $4n - 3 = 177$, $8n - 4 = 356$, and $16n - 7 = 713$.

C. 412 ▱s: No. There is no whole number n that satisfies the equation $4n - 3 = 412$.
412 □s: Yes. The 103rd design will use 412 □s.
412 ◇s: Yes. The 52nd design will use 412 ◇s.
412 blocks: No. There is no whole number n that will satisfy the equation $16n - 7 = 412$.

D. We always have 9 blocks in the center core. We also have 4 arms going out from the center core, with $4(n - 1)$ blocks each. The total is therefore $4 \times [4(n - 1)] + 9 = 4 \times (4n - 4) + 9 = 16n - 16 + 9 = 16n - 7$.

E. ▱: The numbers grow by 4.
□: The numbers grow by 4.
◇: The numbers grow by 8.
Total: The numbers grow by 16.
These are the numbers in front of n in the generalizations.

F. 201, 204, 404
When $16n - 7 = 809$, $n = 51$, $4n - 3 = 201$, $4n = 204$, and $8n - 4 = 404$.

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G. When we combine $4n - 3$, $4n$, and $8n - 4$, we have sixteen n's, or $16n$, and $(-3) + (-4) = -7$ for a total of $16n - 7$.

Pattern 37 Answers

Design	▱	▰	⬡	△	Total
1	1	2	1	0	4
2	2	3	2	1	8
3	3	4	3	2	12
4	4	5	4	3	16
5	5	6	5	4	20
⋮	⋮	⋮	⋮	⋮	⋮
10	10	11	10	9	40
⋮	⋮	⋮	⋮	⋮	⋮
20	20	21	20	19	80
⋮	⋮	⋮	⋮	⋮	⋮
n	n	n + 1	n	n − 1	4n

A. 39, 40, 39, 38
For the 39th design, $n = 39$, $n + 1 = 40$, and $n - 1 = 38$.

B. 78
If $n - 1 = 76$, then $n = 77$ and $n + 1 = 78$.

C. We always have n ▱s and n ⬡s for each design. We also always have $n + 1$ ▰s and $n - 1$ △s, so all together we have $n + n + (n + 1) + (n - 1)$, or $4n$, blocks in all.

D. 368
If $n + 1 = 93$, then $n = 92$ and $4n = 368$.

E. 382 ▰s: Yes. The 381st design will use 382 ▰s.
382 ⬡s: Yes. The 382nd design will use 382 ⬡s.
382 △s: Yes. The 383rd design will use 382 △s.
382 blocks: No. There is no whole number n that will satisfy the equation $4n = 382$.

F. 213, 214, 213, 212
When $4n = 852$, $n = 213$, $n + 1 = 214$, and $n - 1 = 212$.

G. When we combine n, $n + 1$, n, and $n - 1$, we have four n's, or $4n$, plus $(+1) + (-1) = 0$ for a final total of $4n$.

Pattern 38 Answers

Design	△	▱	▰	◇	□	Total
1	1	3	3	6	3	16
2	4	6	9	12	6	37
3	7	9	15	18	9	58
4	10	12	21	24	12	79
5	13	15	27	30	15	90
⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	28	30	57	60	30	205
⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	58	60	117	120	60	415
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	3n − 2	3n	6n − 3	6n	3n	21n − 5

A. 106, 108, 213, 216, 108

B. 130
If $6n - 3 = 261$, then $n = 44$ and $3n - 2 = 130$.

C. The center of each design always contains 16 blocks. From the center extend three arms, each using $n - 1$ clusters of 7 blocks, so each arm uses $7(n - 1)$ blocks. The total will always be $3[7(n - 1)] + 16 = 3(7n - 7) + 16 = 21n - 21 + 16 = 21n - 5$.

D. 327 △s: No. There is no n that satisfies the equation $3n - 2 = 327$.
327 ▱s: Yes. The 109th design will use 327 ▱s.
327 ▰s: Yes. The 55th design will use 327 ▰s.
327 ◇s: No. There is no whole number n that satisfies the equation $6n = 327$.
327 blocks: No. There is no whole number n that satisfies the equation $21n - 5 = 327$.

E. △: The numbers grow by 3.
▱: The numbers grow by 3.
▰: The numbers grow by 6.
◇: The numbers grow by 6.
□: The numbers grow by 3.
Total: The numbers grow by 21.
These are the numbers in front of n in the generalizations.

F. 169, 171, 339, 342, 171
When $21n - 5 = 1192$, $n = 57$, $3n - 2 = 169$, $3n = 171$, $6n - 3 = 339$, and $6n = 342$.

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G. When we combine $3n - 2$, $3n$, $6n - 3$, $6n$, and $3n$, we have twenty-one n's, or $21n$, plus $(-2) + (-3) = -5$ for a total of $21n - 5$.

Pattern 39 Answers

Design	⬡	◇	□	△	Total
1	1	4	1	2	8
2	2	8	2	5	17
3	3	12	3	8	26
4	4	16	4	11	35
5	5	20	5	14	44
⋮	⋮	⋮	⋮	⋮	⋮
10	10	40	10	29	89
⋮	⋮	⋮	⋮	⋮	⋮
20	20	80	20	59	179
⋮	⋮	⋮	⋮	⋮	⋮
n	n	4n	n	3n – 1	9n – 1

A. 38, 152, 38, 113
For design 38, $n = 38$, $4n = 152$, and $3n - 1 = 113$.

B. 140
If $3n - 1 = 104$, then $n = 35$ and $4n = 140$.

C. Each design has n of the basic original shape of 8 blocks for a subtotal of $8n$ blocks. There are also $n - 1$ △s connecting the basic shapes for a total of $8n + (n - 1) = 9n - 1$ blocks.

D. 260 ◇s: Yes. The 65th design will use 260 ◇s.
260 △s: Yes. The 87th design will use 260 △s.
260 blocks: Yes. The 29th design will use 260 blocks.

E. ⬡: The numbers grow by 1.
◇: The numbers grow by 4.
□: The numbers grow by 1.
△: The numbers grow by 3.
Total: The numbers grow by 9.
These are the numbers in front of n in the generalizations.

F. 57, 228, 57, 170
When $9n - 1 = 512$, $n = 57$, $4n = 228$, and $3n - 1 = 170$.

G. When we combine n, $4n$, n, and $3n - 1$, we get nine n's, or $9n$, plus –1 for a total of $9n - 1$.

Pattern 40 Answers

Design	□	▱	△	◇	Total
1	1	4	8	4	17
2	5	16	32	16	69
3	9	28	56	28	121
4	13	40	80	40	173
5	17	52	104	52	225
⋮	⋮	⋮	⋮	⋮	⋮
10	37	112	224	112	485
⋮	⋮	⋮	⋮	⋮	⋮
20	77	232	464	232	1005
⋮	⋮	⋮	⋮	⋮	⋮
n	4n – 3	12n – 8	24n – 16	12n – 8	52n – 35

A. 137, 412, 824, 412
If $n = 35$, then $4n - 3 = 137$, $12n - 8 = 412$, $24n - 16 = 824$, and $12n - 8 = 412$.

B. 728
If $12n - 8 = 364$, then $n = 31$ and $24n - 16 = 728$.

C. 181
If $12n - 8 = 544$, then $n = 46$ and $4n - 3 = 181$.

D. Each design has a center core of 17 blocks. We then add $4(n - 1)$ new clusters, with 13 blocks per cluster. This gives a total of $17 + 4(n - 1)(13) = [17 + 52(n - 1)] = 17 + 52n - 52 = 52n - 35$ blocks.

E. □: The numbers grow by 4.
▱: The numbers grow by 12.
△: The numbers grow by 24.
◇: The numbers grow by 12.
Total: The numbers grow by 52.
These are the numbers before n in the generalizations.

F. 89, 268, 536, 268
When $52n - 35 = 1161$, $n = 23$, $4n - 3 = 89$, $12n - 8 = 268$, and $24n - 16 = 536$.

G. When we combine $4n - 3$, $12n - 8$, $24n - 16$, and $12n - 8$, we have fifty-two n's, or $52n$, plus $(-3) + (-8) + (-16) + (-8) = -35$ for a total of $52n - 35$.

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Pattern 41 Answers

Design	
1	2
2	6
3	12
4	20
5	30
⋮	⋮
10	110
⋮	⋮
20	420
⋮	⋮
n	n(n + 1) or $n^2 + n$

A. 1122
If $n = 33$, then $n^2 + n = 1122$. (Students may experiment with a calculator to find the answer by trial and error.)

B. The basic shape is similar to a rectangle. Each new design is a rectangle with length and width, as represented by the number of blocks, each increased by one.

C. The numbers grow by 4, 6, 8, 10, 12, etc.
No, there is no constant difference between consecutive numbers; a constant difference of 2 occurs when we make a second subtraction, checking the differences between original differences (see table of differences, lower right).

Differences	
Design	
1	2
	> 4
2	6
	> 6
3	12
	> 8
4	20

D. We are making rectangles in which one dimension, the length or the width, is always an even number and the other dimension is an odd number. An even number times an odd number always gives an even number.

E. 37
When $n^2 + n = 1406$, $n = 37$. Students may experiment with a calculator to find the answer by trial and error.

F. The generalization for n is $n(n + 1)$, which is always the dimensions of the rectangle (width and length represented by the numbers of blocks).

G. The generalization is the same as $n(n + 1)$; i.e., $n^2 + n = n(n + 1)$.

Pattern 42 Answers

Design	
1	1
2	4
3	9
4	16
5	25
⋮	⋮
10	100
⋮	⋮
20	400
⋮	⋮
n	n^2

A. 1521
If $n = 39$, then $n^2 = 1521$. (Students may experiment with a calculator to find the answer by trial and error.)

B. These numbers of objects can be arranged in the shape of a square for each design.

C. 47th
If $n^2 = 2209$, then $n = 47$.

D. 2601 ◺s: Yes. The 51st design will use exactly 2601 blocks (2601 is a square number).
3000 ◺s: No. There is no whole number n that will satisfy the equation $n^2 = 3000$.
3969 ◺s: Yes. The 63rd design will use exactly 3969 blocks (3969 is a square number).

E. The numbers grow by 3, 5, 7, 9, 11, 13, etc. There is no constant difference between consecutive numbers. We find a constant difference on the second subtraction, when we check the differences between original differences. That constant difference is 2.

F. $(48)(48) = 48^2 = 2304$
We can see from the samples that you can square the number of odd numbers to get the sum. Therefore, 48×48, or 2304, will equal the sum of the first 48 odd numbers. (Students may use calculators to get the answer by trial and error.)

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Pattern 43 Answers

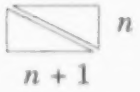
Design	△
1	1
2	3
3	6
4	10
5	15
⋮	⋮
10	55
⋮	⋮
20	210
⋮	⋮
n	$\frac{n(n+1)}{2}$

A. 703

When $n = 37$, $\frac{n(n+1)}{2} = 703$.

B. Using these numbers of objects, each design forms the shape of a triangle (each progressive design forms a larger triangle than the one before).

C. The dimensions of the rectangle (as demonstrated, lower right) will be $n \times (n + 1)$. The area of the rectangle will therefore be $n(n + 1)$. Since the triangle is one-half of the rectangle, the total for the triangle will be $1/2(n)(n + 1)$, or $\frac{n(n+1)}{2}$.



D. The numbers grow by 2, 3, 4, 5, 6, etc. No, there is no constant difference between the numbers the first time; there is a constant difference of 1 when we subtract the original differences.

E. 56

When $\frac{n(n+1)}{2} = 1596$, $n = 56$.

F. 1,275; 20,100; 32,004,000

To get the answers, you could add all the numbers of the sum. A faster way is to use the triangle number formula: $\frac{(50)(51)}{2} = 1275$; $\frac{(200)(201)}{2} = 20{,}100$; $\frac{(8000)(8001)}{2} = 32{,}004{,}000$.

Pattern 44 Answers

Design	<>	⬡	Total
1	1	3	4
2	4	5	9
3	9	7	16
4	16	9	25
5	25	11	36
⋮	⋮	⋮	⋮
10	100	21	121
⋮	⋮	⋮	⋮
20	400	41	441
⋮	⋮	⋮	⋮
n	n^2	2n + 1	$n^2 + 2n + 1$ or $(n + 1)(n + 1)$

A. 1225, 71

If $n = 35$, then $n^2 = 1225$ and $2n + 1 = 71$.

B. The numbers in the <> column and the Total column are the square numbers. They can be arranged in the shape of squares. The numbers in the Total column make the next size larger squares: $(n + 1)^2$, compared to n^2.

C. The numbers in the ⬡ column grow by 2; 2 is the number in front of n in our generalization $2n + 1$.

D. The numbers grow by 3, 5, 7, 9, 11, etc. in the <> column. A constant difference of 2 occurs on the second subtraction (take the difference between original differences). This means the final generalization will start with n^2. (Take 1/2 the constant difference to get the coefficient in front of n.)

E. 2116, 93

When $(n + 1)^2 = 2209$, $n = 46$, $n^2 = 2116$, and $2n + 1 = 93$.

F. We add n^2 and $2n + 1$ to get one n^2, two n's, or $2n$, and 1 for a sum of $n^2 + 2n + 1$.

G. Yes; $(n + 1)^2$ is shown to equal $n^2 + 2n + 1$, as follows:

$$\begin{array}{r} n + 1 \\ \times\ n + 1 \\ \hline n + 1 \\ n^2 + n \\ \hline n^2 + 2n + 1 \end{array}$$

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Pattern 45 Answers

Design	▱	◇	Total
1	2	0	2
2	6	2	8
3	12	6	18
4	20	12	32
5	30	20	50
⋮	⋮	⋮	⋮
10	110	90	200
⋮	⋮	⋮	⋮
20	420	380	800
⋮	⋮	⋮	⋮
n	n^2+n	n^2-n	$2n^2$

A. 1560, 1482
When $n = 39$, $n^2 + n = 1560$ and $n^2 - n = 1482$.

B. For the ▱s, the square always has dimensions of $n \times n$ or n^2 and the column has dimensions of $1 \times n$ or n. Thus, the total is $n^2 + n$.

C. For the ◇s, the shape always is one row short of a square, so the shape has dimensions of $n \times (n - 1)$ or $n^2 - n$.

D. ▱ and ◇: The numbers grow by 2, 4, 6, 8, 10, 12, etc. In the Total column, the numbers grow by 6, 10, 14, 18, 22, 26, etc. The constant difference occurs the second time we take differences; the amount is 2 for the first two columns and 4 for the total column. This means the generalizations will have an n^2 in the first two columns and $2n^2$ in the Total column.

E. 3306, 3192
If $2n^2 = 6498$, then $n^2 = 3249$, $n = 57$, $n + n^2 = 3306$, and $n^2 - n = 3192$.

F. Combining $(n^2 + n) + (n^2 - n)$, we get two n^2s, or $2n^2$, plus $(+ n)$ and $(- n)$ for a total of $2n^2$.

Pattern 46 Answers

Design	△	▱	Total
1	1	3	4
2	4	4	8
3	9	5	14
4	16	6	22
5	25	7	32
⋮	⋮	⋮	⋮
10	100	12	112
⋮	⋮	⋮	⋮
20	400	22	422
⋮	⋮	⋮	⋮
n	n^2	$n+2$	n^2+n+2

A. 1444, 40
If $n = 38$, then $n^2 = 1444$ and $n + 2 = 40$.

B. 625; 4489; 84,681
One way to find the sums is to square the number of odd numbers: $25^2 = 625$, $67^2 = 4489$, $291^2 = 84{,}681$.

C. △: The numbers grow by 3, 5, 7, 9, 11, 13, etc.
▱: The numbers grow by 1.
Total: The numbers grow by 4, 6, 8, 10, 12, 14, etc.
A constant difference of 2 occurs the second time we take differences in the △ column and in the Total column, which means we will have an n^2 in the generalization for these two columns. A constant difference of 1 occurs the first time we take differences in the ▱ column, which gives us n in the generalization.

D. 4225
If $n + 2 = 67$, then $n = 65$ and $n^2 = 4225$.

E. 5929, 79
When $n^2 + n + 2 = 6008$, $n = 77$, $n^2 = 5929$, and $n + 2 = 79$.

F. When we combine n^2 and $n + 2$, we have one n^2, one n, and 2, for a total of $n^2 + n + 2$.

Pattern 47 Answers

A. 2450, 206
If $n = 35$, then $2n^2 = 2450$ and $6n - 4 = 206$.

B. The two squares growing side-by-side each have the dimensions of $n \times n$ or n^2, so the total will be $2n^2$.

C. □: The numbers grow by 6, 10, 14, 18, 22, etc.
▱: The numbers grow by 6.
Total: The numbers grow by 12, 16, 20, 24, 28, etc.
A constant difference of 6 appears right away in the ▱ column. In the □ and Total

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columns, a constant difference occurs the second time we subtract; that difference is 4. We have $2n^2$ in the generalizations for the □ and Total column and $6n$ in the generalization for the ▱ column.

Design	□	▱	Total
1	2	2	4
2	8	8	16
3	18	14	32
4	32	20	52
5	50	26	76
⋮	⋮	⋮	⋮
10	200	56	256
⋮	⋮	⋮	⋮
20	800	116	916
⋮	⋮	⋮	⋮
n	$2n^2$	$6n-4$	$2n^2+6n-4$

D. 170
If $2n^2 = 1682$, then $n^2 = 841$, $n = 29$, and $6n - 4 = 170$.

E. 3362
If $6n - 4 = 242$, then $n = 41$, $n^2 = 1681$, and $2n^2 = 3362$.

F. 4050, 266
When $2n^2 + 6n - 4 = 4316$, $2n^2 = 4050$, $n^2 = 2025$, $n = 45$, and $6n - 4 = 266$.

G. When we combine $2n^2$ with $6n - 4$, we have a total of $2n^2 + 6n - 4$.

Pattern 48 Answers

Design	△	⬡	Total
1	1	3	4
2	3	9	12
3	6	18	24
4	10	30	40
5	15	45	60
⋮	⋮	⋮	⋮
10	55	165	220
⋮	⋮	⋮	⋮
20	210	630	840
⋮	⋮	⋮	⋮
n	$\frac{(n)(n+1)}{2}$	$\frac{3(n)(n+1)}{2}$	$2n(n+1)$

A. 780, 2340
If $n = 39$, then $\frac{n(n+1)}{2} = 780$ and $\frac{3n(n+1)}{2} = 2340$.

B. The numbers of ⬡ are 3 times the numbers of △. (There are always three ⬡s surrounding every △.) The generalization for ⬡ equals $\frac{3n(n+1)}{2}$, which is three times the generalization for △: $\frac{n(n+1)}{2}$.

C. △: The numbers grow by 2, 3, 4, 5, 6, etc.
⬡: The numbers grow by 6, 9, 12, 15, 18, 21, etc.
Total: The numbers grow by 8, 12, 16, 20, 24, 28, etc.
The constant differences, 1 for the △ column, 3 for the ⬡ column, and 4 for the total column, all occur on the second difference. This means the △ answer will contain $\frac{n^2}{2}$, the ⬡ answer will contain $\frac{3n^2}{2}$, and the Total column answer will contain $\frac{4n^2}{2}$.

D. 3384
If $\frac{n(n+1)}{2} = 1128$, then $n = 47$ and $\frac{3n(n+1)}{2} = 3384$.

E. 1711, 5133
When $2n(n + 1) = 6844$, $n(n + 1) = 3422$, $n = 58$, $\frac{n(n+1)}{2} = 1711$, and $\frac{3n(n+1)}{2} = 5133$.

F. Here is a way to show the connection: $\frac{n(n+1)}{2} + \frac{3(n)(n+1)}{2} = \frac{n^2+n+3(n^2+n)}{2} = \frac{n^2+n+3n^2+3n}{2} = \frac{4n^2+4n}{2} = 2n^2 + 2n = 2n(n + 1)$.

Pattern 49 Answers

A. 741, 114, 114
If $n = 38$, then $\frac{n^2+n}{2} = 741$ and $3n = 114$.

B. The numbers increase by 3 in the ◇ column. These are the multiples of 3.

C. The ⬡s form the shape of a triangle. These numbers are the triangle numbers.

D. ⬡: The numbers grow by 2, 3, 4, 5, 6, etc.
△ and ◇: The numbers grow by 3.
Total: The numbers grow by 8, 9, 10, 11, 12, etc.
A constant difference of 1 for the ⬡ and Total columns occurs on the second difference.

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This means the answers for the ⬡ and Total columns will each contain an $\frac{n^2}{2}$. The △ and ⟨⟩ columns each have a constant difference of 3 the first time, which gives the $3n$ in the answer.

Design	⬡	△	⟨⟩	Total
1	1	3	3	7
2	3	6	6	15
3	6	9	9	24
4	10	12	12	34
5	15	15	15	45
⋮	⋮	⋮		⋮
10	55	30	30	115
⋮	⋮	⋮		⋮
20	210	60	60	230
⋮	⋮	⋮		⋮
n	$\frac{n(n+1)}{2}$	3n	3n	$\frac{n^2+13n}{2}$

E. 946, 129
If $3n = 129$, then $n = 43$ and $\frac{n^2+n}{2} = 946$.

F. 77028, 1176
If $3n = 1176$, then $n = 392$ so $\frac{n^2+n}{2} = 77028$.

G. 1378
If $\frac{n^2+13n}{2} = 1690$, then $n = 52$ and $\frac{n^2+n}{2} = 1378$.

H. $\frac{n(n+1)}{2} + 3n + 3n = \frac{n^2+n}{2} + \frac{6n}{2} + \frac{6n}{2} = \frac{n^2+13n}{2}$

Pattern 50 Answers

Design	▱	△	⬡	Total
1	3	3	0	6
2	9	12	1	22
3	18	27	3	48
4	30	48	6	84
5	45	75	10	130
⋮	⋮	⋮		⋮
10	165	300	45	510
⋮	⋮	⋮		⋮
20	630	1200	190	2020
⋮	⋮	⋮		⋮
n	$\frac{3n^2+3n}{2}$	$3n^2$	$\frac{n^2-n}{2}$	$5n^2+n$

A. 2109, 4107, 666
If $n = 37$, then $\frac{3n^2+3n}{2} = 2109$, $3n^2 = 4107$, and $\frac{n^2-n}{2} = 666$.

B. The ⬡s form the shape of a triangle. The numbers in the ⬡ column are the triangle numbers. The nth triangle number can be found by using the generalization $\frac{n(n-1)}{2}$.

C. ▱: The numbers grow by 6, 9, 12, 15, 18, etc,
△: The numbers grow by 9, 15, 21, 27, 33, etc.
⬡: The numbers grow by 1, 2, 3, 4, 5, etc.
Total: The numbers grow by 16, 26, 36, 46, etc.
All constant differences occur the second time (which tells us all answers contain an n^2): The constant difference of 3 in the ▱ column gives us $\frac{3n^2}{2}$; the constant difference of 6 in the △ column gives us $\frac{6n^2}{2}$; the constant difference of 1 in the ⬡ column gives us $\frac{1n^2}{2}$; and the constant difference of 10 in the Total column gives $\frac{10n^2}{2}$.

D. 315, 91
If $3n^2 = 588$, then $n^2 = 196$, $n = 14$, $\frac{3n^2+3n}{2} = 315$, and $\frac{n^2-n}{2} = 91$.

E. 693, 1323, 210
When $5n^2 + n = 2226$, $\frac{3n^2+3n}{2} = 693$, $3n^2 = 1323$, $n^2 = 441$, $n = 21$, and $\frac{n^2-n}{2} = 210$.

F. Since $3n^2$ is the same as $\frac{6n^2}{2}$, we can add as follows:

$$\frac{3n^2+3n}{2} + \frac{6n^2}{2} + \frac{n^2-n}{2} = \frac{3n^2+3n+6n^2+n^2-n}{2} = \frac{10n^2+2n}{2} = 5n^2+n.$$

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